Optimal Transmit Antenna Selection Rule for Interference-Outage Constrained Underlay CR

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***Abstract*—Transmit antenna selection (TAS) is a low hardware complexity multiple antenna technique that exploits spatial diversity to improve the performance of an interference-constrained cognitive radio (CR) system. In the underlay access mode of CR, the choice of the transmit antenna depends on the link between the secondary transmitter (STx) and its receiver, the interference link from the STx to the primary receiver (PRx), and also the interference constraint imposed on the CR system. We propose a novel selection rule called the λ-weighted interference indicator rule for an underlay CR system that is subject to the interference-outage probability constraint, which constrains the probability that the interference power at the PRx exceeds a threshold. This general constraint also encompasses the widely studied peak interference power constraint. We prove that the proposed rule minimizes the average symbol error probability (SEP) of the CR system. It applies to a general class of fading models with continuous probability distributions and many constellations, and outperforms the many selection rules studied in the literature. We analyze its SEP, and present several insights about its novel structure and behavior.**

***Index Terms*—Cognitive radio, underlay, antenna selection, interference-outage constraint, symbol error probability.**

I. Introduction

Cognitive radio (CR) technology has the potential to mitigate the scarcity of radio spectrum by enabling different classes of users to access the spectrum together without an exclusive and inefficient allocation of spectrum to a single class of users [1], [2]. Owing to its promise, it has been incorporated in IEEE standards such as 802.11af and 802.22 [3]. In CR, two categories of users exist, namely, primary users (PUs), who are licensed users of the spectrum, and secondary users (SUs), who are unlicensed users.

Different modes of access have been investigated for the SUs. In the underlay mode, a secondary transmitter (STx) can transmit to a secondary receiver (SRx) concurrently in the same spectrum and at the same time as the PU, but under strict constraints on the interference its transmissions cause to the primary receiver (PRx). Various interference constraints have been investigated in the literature. These include: (i) Peak interference power constraint [4], [5], in which the instantaneous interference power at the PRx cannot exceed a threshold; (ii) Average interference power constraint [6]–[8], in which the fading-averaged interference power at the PRx cannot exceed a threshold; (iii) Primary signal-to-interference-plus-noise-ratio (SINR) constraint [9], in which the SINR at the PRx cannot fall below a threshold; and (iv) Interference-outage constraint [2], [8], in which the event that the interference power exceeds a certain value is referred to as an interference-outage, and the probability of an interference-outage cannot exceed a threshold. These different interference constraints impose different fundamental limits on the performance of the secondary system.

In order to overcome the above limitations of underlay CR, transmit antenna selection (TAS) has attracted considerable attention in the literature [4]–[7], [9]–[11]. It employs only one radio frequency (RF) chain, and dynamically switches it to one of the antennas based on the current channel conditions. It exploits the spatial diversity afforded by having multiple antennas while avoiding the resultant hardware complexity [12]. In underlay CR, the choice of the transmit antenna depends not only on the channel gain from the STx to the SRx (STx-SRx), but also on the channel gain from the STx to the PRx (STx-PRx) in order to ensure compliance with the interference constraint. The interference constraint itself affects the TAS rule employed by the secondary system.

Different TAS rules have been customized in the literature for the peak interference power constraint in [4], [5], [10], [11], for the average interference power constraint in [6], [7], and the primary SINR constraint in [9]. However, the TAS rule for the interference-outage constraint has received less attention. This constraint is of considerable interest by itself both theoretically and practically. Firstly, it is a generalization of the peak interference power constraint, which corresponds to the extreme case in which the interference-outage probability is forced to be zero. Therefore, studying it will result in a richer class of TAS rules. Secondly, it is also justifiable in terms of its impact on the primary system in many scenarios. For example, it is justifiable when the primary system offers delay or disruption tolerant services, or is designed to tolerate deep fades or co-channel interference.

A. Contributions

We make the following contributions:

- We present a novel TAS rule for an interference-outage constrained underlay CR system. We prove that it is optimal in terms of minimizing the fading-averaged symbol error probability (SEP) of the secondary system for a large class of fading models, which have a continuous cumulative distribution function (CDF), and for many constellations. The rule differs from those considered in [4]–[7], [9]–[11].

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- We show that the optimal TAS rule selects an antenna that minimizes the sum of two terms. The first term is the instantaneous SEP and the second term is the indicator function of the STx-PRx channel power gain weighted by an interference-outage penalty factor λ.
- We then analyze the average SEP of the optimal TAS rule. Owing to the novel structure of the rule, its analysis and the final results differ from those for the optimal rule for the average interference constraint, which was studied in [6], and for the peak interference constraint, which was studied in [11]. We first derive an expression for the exact average SEP in a single integral form, and then simplify it to a closed-form for a range of values of λ. We also present an integral-free approximation that applies for all λ, and verify numerically that it is accurate.

B. Outline and Notation

The rest of the paper is organized as follows. Section II presents the system model and the problem statement. The optimal TAS rule, its interference-outage probability, and its SEP are studied in Section III. Numerical results in Section IV are followed by our conclusions in Section V. Mathematical derivations are relegated to the Appendix.

Notation: The absolute value of a complex number x is denoted by $|x|$. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. For a random variable (RV) X, $f_X(\cdot)$ denotes its probability density function (PDF), $F_X^c(\cdot)$ denotes its complementary CDF (CCDF), and $\mathbf{E}_X[\cdot]$ denotes its expectation. Scalar variables are written in normal font and vector variables are written in bold font. The indicator function $I_{\{a\}}$ is 1 if a is true and is 0 otherwise.

II. System Model and Problem Statement

The system model is shown in Figure 1. It consists of an STx that sends data to an SRx, and, in the process, interferes with the PRx. The STx has N_t transmit antennas and one RF chain. It dynamically selects one antenna and connects it to the RF chain for transmission of data. The PRx and SRx are equipped with one receive antenna each. Let the instantaneous channel power gain from the i^{th} antenna of the STx to the SRx antenna be h_i, for $1 \leq i \leq N_t$, and the instantaneous channel power gain from the i^{th} antenna of the STx to the PRx be g_i. Let $\mathbf{h} \triangleq [h_1, \ldots, h_{N_t}]$ and $\mathbf{g} \triangleq [g_1, \ldots, g_{N_t}]$. The STx-SRx channel gains are assumed to be independent and identically distributed (i.i.d.) RVs, and so are the STx-PRx channels. This is justified when the antennas are sufficiently spaced apart, and has been assumed in [4], [6], [10].

A. Antenna Selection Options and Data Transmission

The STx transmits a data symbol d that is drawn with equal probability from a constellation of M symbols. It selects one out of N_t antennas and transmits with a fixed power P_t. Further, it can also decide to transmit with none of the antennas, i.e., it transmits with zero-power, in order to not interfere at all with the PRx. For ease of presentation, we shall denote this as a transmission from a virtual antenna 0, and define the corresponding channel power gains as $h_0 \triangleq 0$ and $g_0 \triangleq 0$. Let $s \in \{0, 1, \ldots, N_t\}$ denote the antenna selected.

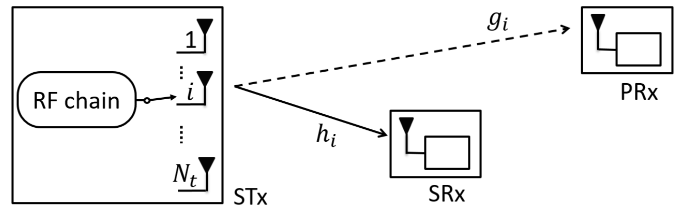

Fig. 1. System model showing a secondary system that contains an STx with N_t transmit antennas and one RF chain. It transmits data to an SRx and causes interference to a PRx.

Let the signal received at the SRx be r_s, the interference signal at SRx due to transmissions by the primary transmitter (PTx) be i_s, and the interference at the PRx due to transmissions by the STx be i_p. Then, r_s and i_p are given by

$$r_s = \sqrt{P_t}\sqrt{h_s}e^{j\theta_s}d + n + i_s, \quad (1)$$

$$i_p = \sqrt{P_t}\sqrt{g_s}e^{j\varphi_s}d, \quad (2)$$

where $\mathbf{E}\left[|d|^2\right] = 1$, θ_s and φ_s are the phases of the complex baseband STx-SRx and STx-PRx channel gains, respectively, and n is a circular symmetric complex additive white Gaussian noise. We assume i_s to be Gaussian. Therefore, $n + i_s$ is a circular symmetric complex Gaussian RV, whose variance is denoted by σ^2. The Gaussian interference assumption at the SRx corresponds to a worst case model for interference, and is widely assumed either explicitly or implicitly in the literature to ensure tractability [6]–[8]. Its validity depends on the characteristics of the channel from the PTx to the SRx and signal transmitted by the PTx [2], [8]. We refer the reader to [8] for a detailed discussion and comparison of alternate approaches for modeling i_s.

Channel State Information (CSI) Assumptions and Justifications: In order to perform TAS, we make the classical assumption that the STx knows the STx-SRx channel power gains $\mathbf{h}$ [4]–[6], [10]. However, the STx does not know any channel phase information. The SRx uses a coherent demodulator. Hence, it only knows h_s and θ_s. These can be estimated by inserting a pilot symbol along with the data. Also, the STx, but not the SRx, knows the STx-PRx channel power gains $\mathbf{g}$, as also has been widely assumed in the underlay CR literature [4]–[7], [10]. In time division duplexing, $\mathbf{g}$ can be acquired by periodically sensing transmissions by the PRx and exploiting reciprocity [13]. Instead, in frequency division duplexing, it can be obtained by using a *hidden power-feedback* loop technique [14], without requiring any feedback from the PRx.

B. Problem Statement

A TAS rule $\phi : (\mathbb{R}^+)^{N_t} \times (\mathbb{R}^+)^{N_t} \rightarrow \{0, 1, \ldots, N_t\}$ is a mapping from $(\mathbf{h}, \mathbf{g})$ to the set of $N_t + 1$ available transmit antennas. Let $\text{SEP}(h_s)$ denote the instantaneous SEP using the selected antenna s. From [15, (14)], it is given by

$$\mathrm{SEP}(h_s) \approx c_1 \exp\left(-\frac{P_t h_s}{c_2 \sigma^2}\right), \quad \text{for} \quad 0 \le s \le N_t, \tag{3}$$

where c_1 and c_2 are modulation-specific constants. For example, they are 0.6 and 5.5 for 8-PSK and 0.8 and 8.2 for 16-QAM. While (3) is an approximation, it applies to many constellations such as MPSK, MQAM, differential PSK (DPSK), and M-ary frequency-shift-keying (MFSK) [16]. It is accurate and tractable. It also implies that for $s = 0$, the SEP is equal to its largest value c_1. This can be interpreted as a penalty for choosing not to transmit, and ensures that the trivial policy in which the STx never transmits is not optimal.[1]

Our goal is to find the optimal TAS rule ϕ^* among the space of all TAS rules $\mathcal{D}$, that minimizes the average SEP of the SU under the interference-outage probability constraint. Formally, the optimal TAS rule is the solution of the following stochastically constrained optimization problem $\mathcal{P}$:

$$\mathcal{P}: \quad \min_{\phi \in \mathcal{D}} \quad \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\mathrm{SEP}(h_s)\right] \tag{4}$$

$$\text{s.t.} \quad \Pr\left(P_t g_s > \tau\right) \le O_{\max}, \tag{5}$$

$$s = \phi(\mathbf{h}, \mathbf{g}), \tag{6}$$

where $\Pr\left(P_t g_s > \tau\right)$ is the probability that the interference power at the PRx exceeds an interference power threshold τ and $O_{\max}$ is the maximum value that is allowed for it. We focus on minimizing the average SEP as it is an important performance measure of a communication system, and has attracted considerable interest in the literature [6], [8].

III. Optimal TAS Rule and its Performance

We now propose a new TAS rule called the *λ-weighted interference indicator rule* and characterize its properties. We derive its interference-outage probability and use it to prove that it solves the problem $\mathcal{P}$ above for a general class of fading models. We then analyze its average SEP for Rayleigh fading.

A. λ-Weighted Interference Indicator Rule

Let

$$y_i \triangleq \exp\left(-\frac{P_t h_i}{c_2 \sigma^2}\right), \quad \text{for} \quad 0 \le i \le N_t, \tag{7}$$

and let $\mathbf{y} \triangleq [y_1, \ldots, y_{N_t}]$. We define the following parametric TAS rule $\mathcal{S}_\lambda$, which we shall call as the *λ-weighted interference indicator* rule, in terms of a parameter $\lambda \in [0, c_1]$:

$$\mathcal{S}_\lambda: \quad s = \mathop{\arg\min}_{i \in \{0,1,\ldots,N_t\}} \left\{ y_i + \frac{\lambda}{c_1} I_{\left\{g_i > \frac{\tau}{P_t}\right\}} \right\}. \tag{8}$$

To help understand the behavior of $\mathcal{S}_\lambda$, we introduce the following terminology. We shall refer to $y_i + \frac{\lambda}{c_1} I_{\left\{g_i > \frac{\tau}{P_t}\right\}}$ as the *selection metric* of antenna i. Thus, $\mathcal{S}_\lambda$ selects the antenna with the lowest selection metric. We shall refer to an antenna i for which $g_i \le \frac{\tau}{P_t}$ as an *outage-compatible* antenna. Otherwise, it as an *outage-incompatible* antenna. Clearly, antenna zero is always outage-compatible and its selection metric is $y_0 = 1$. We now explain how $\mathcal{S}_\lambda$ behaves for different values of λ.

[1]Even with zero transmit power an SEP of $m \triangleq 1 - (1/M) < 1$ can be achieved for a constellation of size M. This happens, for example, when the SRx declares one among the M points in the constellation as the transmitted point.

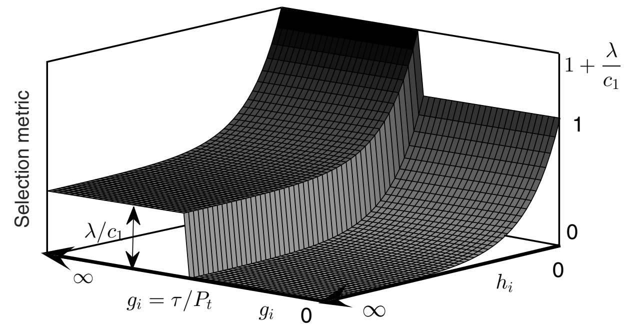

Fig. 2. Selection metric of antenna i as a function h_i and g_i.

(i) For $\lambda = 0$, the selection metric of antenna i is y_i. Using the monotonicity of y_i, $\mathcal{S}_0$ is equivalent to:

$$\mathcal{S}_0: \quad s = \mathop{\arg\max}_{i \in \{1,2,\ldots,N_t\}} \left\{h_i\right\}. \tag{9}$$

From (3) and (4), $\mathcal{S}_0$ is clearly the optimal TAS rule if the interference-outage constraint in (5) is not present. Its interference-outage probability is given by

$$U = \Pr\left(g_s > \frac{\tau}{P_t}\right) = \Pr\left(g_1 > \frac{\tau}{P_t}\right). \tag{10}$$

The second equality above follows because the antenna s selected by $\mathcal{S}_0$ does not depend on g_s, and $g_1, \ldots, g_{N_t}$ are i.i.d.

(ii) For $\lambda > 0$, the selection metric is a linear combination of an exponentially decreasing function of h_i and the indicator function $I_{\left\{g_i > \frac{\tau}{P_t}\right\}}$. A three-dimensional view of the selection metric as a function of h_i and g_i is shown in Figure 2. Notice its discontinuous behavior with respect to g_i.

(iii) For $\lambda = c_1$, the selection metric of every outage-incompatible antenna is greater than $y_0 = 1$. Consequently, $\mathcal{S}_\lambda$ selects the outage-compatible antenna with the highest STx-SRx channel power gain, i.e., the outage-compatible antenna with the lowest instantaneous SEP. Therefore, only for this case, it is equivalent to the rule proposed in [4] for the peak interference power constraint.

B. Optimal TAS Rule

We first derive the interference-outage probability of $\mathcal{S}_\lambda$. Since $y_1, \ldots, y_{N_t}$ are i.i.d., we denote their marginal CCDF and PDF by $F_y^c(\cdot)$ and $f_y(\cdot)$, respectively.

Lemma *1:* The interference-outage probability O_λ of $\mathcal{S}_\lambda$, for $0 \le \lambda \le c_1$, is given by

$$O_\lambda = N_t U \int_0^{1-\frac{\lambda}{c_1}} \left[U F_y^c(x) + (1-U) F_y^c\left(x + \frac{\lambda}{c_1}\right) \right]^{N_t - 1} f_y(x)\, dx. \tag{11}$$

Proof: The proof is given in Appendix A. ■

Insights: Replacing $x + \frac{\lambda}{c_1}$ with $\frac{\lambda}{c_1}$ in (11), the following closed-form upper bound for O_λ can be derived:

$$O_\lambda \leq \left[U + (1-U)\, F_y^c\left(\frac{\lambda}{c_1}\right)\right]^{N_t} - \left[U F_y^c\left(1 - \frac{\lambda}{c_1}\right) + (1-U)\, F_y^c\left(\frac{\lambda}{c_1}\right)\right]^{N_t}. \quad (12)$$

For the class of continuous fading distributions, it follows directly from Lemma 1 that O_λ is a continuous and strictly monotonically decreasing function of λ.

Using Lemma 1, we prove the following key result.

Theorem *1:* If $O_{\max} \geq U$, the optimal TAS rule is given by $\mathcal{S}_0$. Else, for $0 \leq O_{\max} < U$, it is given by $\mathcal{S}_{\lambda^*}$, where $\lambda^* > 0$ is the solution of $O_\lambda = O_{\max}$. Such a choice of λ^* is unique, strictly positive, and always exists.

Proof: The proof is given in Appendix B. ■

Notice that the $\mathcal{S}_{\lambda^*}$ only needs to know if g_i exceeds a threshold τ/P_t. This makes it robust against the estimation errors of the STx-PRx channel power gain.

C. SEP Analysis of the Optimal TAS Rule for Rayleigh Fading

Next, we derive the average SEP expression of $\mathcal{S}_{\lambda^*}$ for Rayleigh fading. Here, h_i and g_i are i.i.d. exponential RVs. Let μ_h and μ_g denote the means of h_i and g_i, respectively. It can be shown that (10) simplifies to $U = \exp\left(-\tau/(\mu_g P_t)\right)$. Therefore, the optimal TAS rule will be the same as $\mathcal{S}_0$ for $\tau \geq P_t \mu_g \ln\left(1/O_{\max}\right)$. Let $\Omega \triangleq P_t \mu_h/\sigma^2$ denote the mean signal-to-noise-ratio of the secondary system and $m \triangleq 1 - (1/M)$.

Theorem *2:* The average SEP of the secondary system denoted by the $\overline{\text{SEP}}$, for the optimal TAS rule $\mathcal{S}_{\lambda^*}$ is given by

$$\overline{\text{SEP}} = m U^{N_t}\left[1 - \left(1 - \frac{\lambda^*}{c_1}\right)^{\frac{c_2}{\Omega}}\right]^{N_t} + T_2, \quad (13)$$

where

$$T_2 = \frac{N_t c_1 c_2}{\Omega} \sum_{k=0}^{N_t-1} \sum_{i=0}^{k} \sum_{j=0}^{N_t-k-1} \binom{N_t-1}{k}\binom{k}{i}\binom{N_t-k-1}{j} \times (-1)^{i+j} U^k (1-U)^{N_t-k-1}\left(U\psi_{j,i+1} + (1-U)\psi_{j+1,i}\right) + \frac{N_t U^{N_t} c_2 \lambda^*}{\Omega} \sum_{k=0}^{N_t-1}\sum_{n=0}^{k} \frac{\binom{N_t-1}{k}\binom{k}{n}(-1)^n\left(\frac{1}{U}-1\right)^{k+1}}{\left(\frac{c_2(n+1)}{\Omega}+1\right)\left(\frac{\lambda^*}{c_1}\right)^{-\frac{c_2(n+1)}{\Omega}}}, \quad (14)$$

and $\psi_{k_1,k_2} = \int_0^{1-\frac{\lambda^*}{c_1}} \left(x + \frac{\lambda^*}{c_1}\right)^{\frac{c_2 k_1}{\Omega}} x^{\frac{c_2 k_2}{\Omega}}\, dx$.

Proof: The proof is given in Appendix C. ■

In general, ψ_{k_1,k_2} can be computed accurately as a sum of n terms as follows:

$$\psi_{k_1,k_2} = \left[1 - \frac{\lambda^*}{c_1}\right]^{\frac{c_2 k_2}{\Omega}+1} \sum_{i=1}^{n} w_i \left(z_i\left[1 - \frac{\lambda^*}{c_1}\right] + \frac{\lambda^*}{c_1}\right)^{\frac{c_2 k_1}{\Omega}} z_i^{\frac{c_2 k_2}{\Omega}} + \epsilon_n, \quad (15)$$

where z_i and w_i are the n Gauss quadrature abscissas and weights [17], respectively. The error term ϵ_n decreases as $O(1/n^2)$ [18]. For the parameters of interest, even $n = 4$ terms turn out to be sufficient. Furthermore, for $\lambda^* \in (c_1/2, c_1]$, the integral can be written in terms of a series as follows [19]:

$$\psi_{k_1,k_2} = \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{c_2 k_1}{\Omega}+1\right)\left(\frac{\lambda^*}{c_1}\right)^{\frac{c_2 k_1}{\Omega}-k}\left(1 - \frac{\lambda^*}{c_1}\right)^{\frac{c_2 k_2}{\Omega}+k+1}}{\Gamma\left(\frac{c_2 k_1}{\Omega}-k+1\right) k!\left(\frac{c_2 k_2}{\Omega}+k+1\right)}.$$

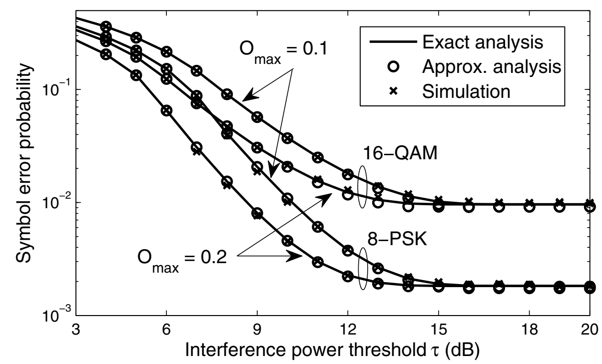

Fig. 3. SEP as a function of τ for 8-PSK and 16-QAM for different values of $O_{\max}$ ($P_t = 13$ dB and $N_t = 8$).

IV. Numerical Results and Performance Benchmarking

We now present Monte Carlo simulations that use 10^6 data symbols to verify our analytical results for Rayleigh fading. We also benchmark the performance of the optimal TAS rule with different TAS rules proposed in the literature. We set $\sigma^2 = 1$ and $\mu_h = \mu_g = 1$.

Figure 3 plots average SEP as a function of τ for $P_t = 13$ dB and $N_t = 8$. This is done for 8-PSK and 16-QAM, and for two values of $O_{\max}$. Results from the SEP formula in (13) and the approximate one in (15) (with $n = 4$ for 16-QAM and $n = 8$ for 8-PSK) are shown. We observe a good match between them and bit-level simulations, which simulate the entire transmit and receive chains and do not assume the formula in (3). For both constellations, the average SEP curves saturate for $\tau \geq P_t \mu_g \ln\left(1/O_{\max}\right) = 16.6$ dB when $O_{\max} = 0.1$ and for $\tau \geq 15$ dB when $O_{\max} = 0.2$.

Performance Benchmarking: We now compare the performance of the optimal TAS rule with the enhanced minimum interference (EMI) rule [6], the difference selection (DS) rule [7], and enhanced maximum-signal-power to leak-interference-power-ratio (EMSLIR) rules [6]. The EMI rule selects the antenna with the lowest STx-PRx channel power gain. However, it selects antenna zero when all the STx-PRx channel power gains are above a threshold, which is chosen to meet interference constraint. The EMSLIR rule selects the antenna with the largest ratio of the STx-SRx channel power gain to the STx-PRx channel power gain. However, it selects antenna zero when the ratios of the channel power gains of all the antennas are below a threshold, which is chosen to meet the interference constraint with equality. It is a relevant

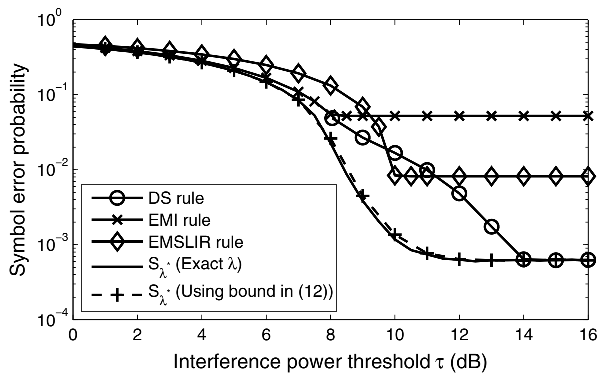

Fig. 4. SEP comparison of the optimal TAS rule and benchmark rules ($N_t = 4$, $P_t = 12$ dB, $O_{\max} = 0.2$, $c_1 = 0.5$, $c_2 = 1.7$ and QPSK).

benchmark as it maximizes the SU capacity under the peak interference power constraint [5]. The DS rule chooses the antenna that maximizes the difference $\delta h_i - (1-\delta)g_i$, where δ is chosen to meet the interference constraint with equality.

Figure 4 compares the average SEP of the optimal TAS rule with that of the EMI, DS, and EMSLIR rules, for $N_t = 4$, $P_t = 12$ dB, and $O_{\max} = 0.2$. (i) For $\tau < 14$ dB, the interference-outage constraint is active for the optimal TAS rule. Here, $\lambda^* > 0$ and the optimal TAS rule outperforms all the benchmark rules. For example, at $\tau = 11$ dB, its average SEP is lower by a factor of 70, 13, and 11 than the EMI, DS, and EMSLIR rules. (ii) For $\tau \geq 14$ dB, the average SEP saturates for both the optimal rule and the DS rule. This is because the interference-outage constraint becomes inactive and they both reduce to $\mathcal{S}_0$ in (9). We also observe that the average SEPs of the EMI and EMSLIR rules saturate for τ greater than 8.5 dB and 11 dB, respectively. We also plot the average SEP of the optimal TAS rule using the value of λ^* that is obtained by equating the integral-free interference-outage probability upper bound in (12) to $O_{\max}$. We see that the degradation in the average SEP is negligible when compared to using the exact λ^*.

V. Conclusions

We proposed a novel TAS rule that minimized the average SEP of an underlay CR that was subject to an interference-outage constraint. We showed that it was optimal for a general class of fading models and for many constellations. We saw that the selection metric of an antenna was a discontinuous function of the STx-PRx channel power gain, and was unlike the many other TAS rules studied in the literature. We derived an exact expression and an integral-free tight approximation for the average SEP of the CR system for Rayleigh fading. We showed that the proposed rule outperformed several TAS rules. Interesting avenues for future work include generalizing the model to allow for multiple PRxs, multiple antennas at the SRx, antenna subset selection, and imperfect CSI.

Appendix

A. Brief Proof of Lemma 1

An interference-outage, i.e., $P_t g_s > \tau$, occurs only when $s \neq 0$. Thus, using the law of total probability and symmetry, we can show that the interference-outage probability $O_\lambda = \Pr\left(g_s > \frac{\tau}{P_t}\right)$ is given by

$$O_\lambda = \sum_{i=1}^{N_t} \Pr\left(s = i, g_i > \frac{\tau}{P_t}\right) = N_t \Pr\left(s = 1, g_1 > \frac{\tau}{P_t}\right). \quad (16)$$

Let k antennas out of the antennas $2, \ldots, N_t$ be outage-compatible. The total number of ways in which such k antennas can be chosen from the $N_t - 1$ antennas is $\binom{N_t-1}{k}$. One such combination is denoted by the set $B_1 = \left\{g_2 \leq \frac{\tau}{P_t}, \ldots, g_{k+1} \leq \frac{\tau}{P_t}, g_{k+2} > \frac{\tau}{P_t}, \ldots, g_{N_t} > \frac{\tau}{P_t}\right\}$. Let $A_1 \triangleq \left\{\left(g_1 > \frac{\tau}{P_t}\right) \cap B_1\right\}$. Given k, all these combinations are equally likely as $g_1, \ldots, g_{N_t}$ are i.i.d. Therefore, using the law of total probability, we can write O_λ in (16) as

$$O_\lambda = N_t \sum_{k=0}^{N_t-1} \binom{N_t-1}{k} \Pr(A_1) \Pr\left(s = 1 \,\middle|\, A_1\right). \quad (17)$$

Since $g_1, \ldots, g_{N_t}$ are i.i.d., we get $\Pr(A_1) = (1-U)^k U^{N_t-k}$.

Given A_1, $\mathcal{S}_\lambda$ selects antenna 1 if $y_1 + \frac{\lambda}{c_1} < y_i$, for $2 \leq i \leq k+1$, $y_1 + \frac{\lambda}{c_1} < y_j + \frac{\lambda}{c_1}$, for $k+2 \leq j \leq N_t$, and $y_1 + \frac{\lambda}{c_1} < y_0 = 1$. Hence,

$$\Pr\left(s=1 \middle| A_1\right) = \Pr\Bigg(y_1 + \frac{\lambda}{c_1} < y_2, \ldots, y_1 + \frac{\lambda}{c_1} < y_{k+1}, y_1 + \frac{\lambda}{c_1} < y_{k+2} + \frac{\lambda}{c_1}, \ldots, y_1 + \frac{\lambda}{c_1} < y_{N_t} + \frac{\lambda}{c_1}, y_1 + \frac{\lambda}{c_1} < 1,\Bigg).$$

Conditioning on y_1, the events in the above probability term are mutually independent. Combining this with the fact that $y_2, \ldots, y_{N_t}$ are i.i.d., we get $\Pr\left(s=1 \,\middle|\, A_1, y_1 = x\right) = F_y^c\left(x + \frac{\lambda}{c_1}\right)^k F_y^c(x)^{N_t-k-1} I_{\left\{x < 1 - \frac{\lambda}{c_1}\right\}}$. Averaging this over y_1 yields the expression for $\Pr\left(s=1 \,\middle|\, A_1\right)$. Substituting this in (17) yields the desired expression in (11).

B. Proof of Theorem 1

We consider the two cases $O_{\max} \geq U$ and $O_{\max} < U$ separately below.

1. $O_{max} \geq U$: Here, $\mathcal{S}_0$ satisfies (5). Hence, from the discussion for the $\lambda = 0$ case in Section III-A, it is optimal.

2. $0 \leq O_{max} < U$: Here, $\mathcal{S}_0$ does not satisfy (5). Therefore, it cannot even be considered. Instead, consider the TAS rule $\mathcal{S}_{\lambda^*}$, where λ^* is chosen such that the interference-outage probability of $\mathcal{S}_{\lambda^*}$ is equal to $O_{\max}$. The existence and uniqueness of $\lambda^* \in [0, c_1]$ follow from the intermediate value theorem [20], since (i) $O_\lambda = U$ when $\lambda = 0$ and $O_\lambda = 0$ when $\lambda = c_1$, and (ii) from Lemma 1, we know that O_λ is a continuous and monotonically decreasing function of λ.

Among all the selection rules, by definition, $\mathcal{S}_{\lambda^*}$ selects an antenna that minimizes $y_i + \frac{\lambda^*}{c_1} I_{\left\{g_i > \frac{\tau}{P_t}\right\}}$ for any given realization of $y_1,\ldots,y_{N_t}$ and $g_1,\ldots,g_{N_t}$. Let s^* be the antenna selected by the $\mathcal{S}_{\lambda^*}$. Therefore, for any TAS rule ϕ with $s \triangleq \phi(\mathbf{h},\mathbf{g})$, it follows that

$$\mathbf{E}_{\mathbf{y},\mathbf{g}}\left[y_{s^*}+\frac{\lambda^*}{c_1}I_{\left\{g_{s^*}>\frac{\tau}{P_t}\right\}}\right] \leq \mathbf{E}_{\mathbf{y},\mathbf{g}}\left[y_s+\frac{\lambda^*}{c_1}I_{\left\{g_s>\frac{\tau}{P_t}\right\}}\right].$$

Taking the expectations inside, we get

$$\mathbf{E}_{\mathbf{y},\mathbf{g}}\left[y_{s^*}\right]+\frac{\lambda^*}{c_1}\mathrm{Pr}\left(g_{s^*}>\frac{\tau}{P_t}\right) \leq \mathbf{E}_{\mathbf{y},\mathbf{g}}\left[y_s\right]+\frac{\lambda^*}{c_1}\mathrm{Pr}\left(g_s>\frac{\tau}{P_t}\right).$$

We know that $\mathrm{Pr}\left(g_{s^*} > \frac{\tau}{P_t}\right) = O_{\max}$. Substituting $y_i = \mathrm{SEP}(h_i)/c_1$ and rearranging terms, we get

$$\mathbf{E}_{\mathbf{h},\mathbf{g}}[\mathrm{SEP}(h_{s^*})] \leq \mathbf{E}_{\mathbf{h},\mathbf{g}}[\mathrm{SEP}(h_s)]+\lambda^*\left[\mathrm{Pr}\left(g_s>\frac{\tau}{P_t}\right)-O_{\max}\right].$$

If ϕ satisfies the constraint in (5), the above inequality implies that $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\mathrm{SEP}(h_{s^*})\right] \leq \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\mathrm{SEP}(h_s)\right]$. Thus, $\mathcal{S}_{\lambda^*}$ is optimal among all the TAS rules that satisfy the interference-outage constraint.

C. Brief Proof of Theorem 2

Let $\mathrm{Pr}\left(\mathrm{Err}|\mathbf{y},\mathbf{g}\right)$ denote the probability of error conditioned on $\mathbf{y}$ and $\mathbf{g}$. We know that $\overline{\mathrm{SEP}} = \mathbf{E}_{\mathbf{y},\mathbf{g}}\left[\mathrm{Pr}\left(\mathrm{Err}|\mathbf{y},\mathbf{g}\right)\right]$. Using the law of total probability and symmetry, we get

$$\begin{aligned}\overline{\mathrm{SEP}} &= \mathbf{E}_{\mathbf{y},\mathbf{g}}\left[\mathrm{Pr}\left(\mathrm{Err}|\mathbf{y},\mathbf{g},s=0\right)\mathrm{Pr}\left(s=0|\mathbf{y},\mathbf{g}\right)\right] \\ &\quad + N_t\mathbf{E}_{\mathbf{y},\mathbf{g}}\left[\mathrm{Pr}\left(\mathrm{Err}|\mathbf{y},\mathbf{g},s=1\right)\mathrm{Pr}\left(s=1|\mathbf{y},\mathbf{g}\right)\right]. \quad (18)\end{aligned}$$

Using $\mathrm{Pr}\left(\mathrm{Err}|\mathbf{y},\mathbf{g},s=0\right) = m$, $\mathrm{Pr}\left(\mathrm{Err}|\mathbf{y},\mathbf{g},s=1\right) = c_1 y_1$, and the law of total expectation, we get[2]

$$\overline{\mathrm{SEP}} = m\mathrm{Pr}\left(s=0\right) + N_t c_1 \mathbf{E}_{y_1}\left[y_1 \mathrm{Pr}\left(s=1|y_1\right)\right].$$

We denote the two terms in the above equation as T_1 and T_2, and evaluate them separately below.

First Term T_1: From the selection rule in (8), we know that $s=0$ is selected when all the antennas $1,\ldots,N_t$ are outage-incompatible and their selection metrics exceed one. Therefore, T_1 is equal to $m\mathrm{Pr}\left(g_1>\frac{\tau}{P_t},\ldots,g_{N_t}>\frac{\tau}{P_t},y_1+\frac{\lambda^*}{c_1}>1,\ldots,y_{N_t}+\frac{\lambda^*}{c_1}>1\right)$. Now, substituting the CCDF of y_1, which is $F_y^c(x) = 1-x^{\frac{c_2}{\Omega}}$, for $x \in (0,1]$, and the CCDF of g_1 in the above formula, we get first term in (13).

Second Term T_2: From the law of total probability, we have $\mathrm{Pr}(s=1|y_1) = \mathrm{Pr}\left(s=1, g_1 > \frac{\tau}{P_t}\,\middle|\, y_1\right) + \mathrm{Pr}\left(s=1, g_1 \leq \frac{\tau}{P_t}\,\middle|\, y_1\right)$.

Let $A_2 \triangleq \left\{\left(g_1 \leq \frac{\tau}{P_t}\right) \cap B_1\right\}$. Summing over all the events in which k antennas out of the antennas $2,\ldots,N_t$ are outage-compatible, we get the following expression for $\mathrm{Pr}\left(s=1|y_1\right)$. Its derivation along lines similar to (17).

$$\begin{aligned}\mathrm{Pr}\left(s=1|y_1\right) &= \sum_{k=0}^{N_t-1}\binom{N_t-1}{k}\mathrm{Pr}\left(s=1\,\middle|\,A_1,y_1\right)\mathrm{Pr}\left(A_1\right) \\ &\quad + \sum_{k=0}^{N_t-1}\binom{N_t-1}{k}\mathrm{Pr}\left(s=1\,\middle|\,A_2,y_1\right)\mathrm{Pr}\left(A_2\right). \quad (19)\end{aligned}$$

[2]Instead of c_1, we use the more accurate value of $m = 1-(1/M)$ for the SEP with zero transmit power.

Recall the definition of the event $A_1 = \left\{\left(g_1 > \frac{\tau}{P_t}\right) \cap B_1\right\}$ from Appendix A. The expression for $\mathrm{Pr}\left(s=1|A_1,y_1=x\right)$ was derived in Appendix A. Along the similar lines it can be shown that $\mathrm{Pr}\left(s=1|A_2,y_1=x\right) = F_y^c(x)^k \times \left[I_{\left\{x \leq \frac{\lambda^*}{c_1}\right\}} + I_{\left\{x>\frac{\lambda^*}{c_1}\right\}}F_y^c\left(x-\frac{\lambda^*}{c_1}\right)^{N_t-k-1}\right]$. Substituting these in (19) yields the expression for $\mathrm{Pr}\left(s=1|y_1\right)$. Lastly, averaging over y_1, substituting $F_y^c(x)$ and $f_y(x) = c_2 x^{\frac{c_2}{\Omega}-1}/\Omega$, for $x \in (0,1]$ yields the expression for T_2 in (14).

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