Optimal Antenna Selection and Power Adaptation for Underlay Spectrum Sharing with Statistical CSI

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***Abstract*—For underlay spectrum sharing, transmit antenna selection is a low hardware complexity technique that can help the secondary system overcome the performance limitations imposed by the constraints on the interference it causes to a primary system. However, its efficacy depends on the channel state information (CSI) available to the secondary transmitter. We consider a practically appealing model in which the secondary transmitter has only statistical CSI about the channel gains from itself to the primary receiver and is subject to a general class of stochastic interference constraints. We derive an optimal and novel joint antenna selection and continuous power adaptation rule for it that minimizes the average symbol error probability (SEP) of the secondary system. We show that it has an intuitively appealing separable structure. We then analyze its average SEP. Our numerical results evaluate the impact of the interference constraint on both secondary and primary systems, and show that a judicious choice of the interference constraint and its parameters is needed as its impact on the secondary and primary systems can be very different.**

I. Introduction

The ever-increasing demand for high wireless data rates and the shortage of spectrum has spawned the development of different spectrum sharing techniques such as cognitive radio, device-to-device communications, and cognitive radio-inspired non-orthogonal multiple access [1]. Spectrum sharing has been adopted in standards such as IEEE 802.11af, long term evolution (LTE)-license assisted access, MulteFire, and citizen's broadband radio service [2], [3]. Next generation wireless standards such as 5G new radio (NR) unlicensed and IEEE 802.11be are also being designed to share the spectrum allocated to primary users (PUs), such as satellite services, with secondary users (SUs) [4].

In the underlay spectrum sharing mode, which is the focus of this paper, a secondary transmitter (STx) transmits simultaneously with the primary transmitter (PTx) under constraints on the interference it causes to the primary receiver (PRx) [5], [6]. However, the interference constraint can severely limit the secondary performance. Multi-antenna techniques exploit spatial diversity to address this limitation. Transmit antenna selection (TAS) is one such technique whose hardware complexity is comparable to a single antenna system [7]–[10]. In it, the STx dynamically selects one among multiple antennas based on channel conditions, connects it to a single radio frequency (RF) chain, and transmits data to the secondary receiver (SRx).

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Channel state information (CSI) at the STx plays a crucial role in determining the efficacy of TAS. In order to adhere to the interference constraint, the STx ideally needs to know the instantaneous channel gains from the STx to the SRx (STx-SRx) and from the STx to the PRx (STx-PRx). Assuming that such instantaneous CSI is available, [7] and [11] developed the TAS rules for an STx that employs on-off power adaptation and transmits with either fixed power or zero power. TAS rules for an STx that employs continuous power adaptation were developed in [8]–[10].

However, acquiring instantaneous CSI of the STx-PRx links at the STx is practically challenging. It requires feedback from the PRx, which entails coordination between the primary and secondary systems, or requires the PRx to transmit periodically and often so that the STx can exploit reciprocity and estimate the instantaneous STx-PRx channel gains in a timely manner. This makes TAS with only statistical CSI of the STx-PRx links practically appealing. It requires a different set of TAS rules compared to those in [7]–[10]. For an STx that uses on-off power adaptation and has only statistical CSI of the STx-PRx links, [6] developed a TAS rule for an interference-outage constrained secondary system. TAS for an STx that transmits with a fixed power and is subject to the average interference constraint was studied in [9], [12].

A. Focus and Contributions

In this paper, we derive the optimal joint TAS and continuous power adaptation (TAS-CPA) rule for an STx that has only statistical CSI of the STx-PRx links and is subject to a general stochastic interference constraint. This constraint, which imposes limits on a fading-averaged instantaneous interference penalty, includes as special cases the average interference constraint and its generalizations [13] and the interference-outage constraint [11]. It differs from the peak interference constraint, which limits the instantaneous interference at the PRx, and can be imposed on secondary systems that have either imperfect or statistical CSI. Another point to note is that the peak interference constraint is conservative not just for secondary systems but also primary systems [14]. Our study with this general and practical approach is timely given that co-existence mechanisms, which includes the specification of the interference constraint, between secondary and primary users in the upcoming standards such as 5G NR unlicensed and IEEE 802.11be are a work in progress [4].

We make the following specific contributions:

1) We derive an optimal TAS-CPA rule that minimizes the average symbol error probability (SEP) of a secondary

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system in which the STx only has the statistical CSI of the STx-PRx links and is subject to a stochastic interference constraint. We focus on the SEP as it is a widely used performance measure in communication systems. We show that the optimal rule has an appealing and intuitive separable structure. Specifically, we show that with statistical CSI, the optimal transmit antenna is independent of the interference constraint for any stochastic interference constraint, which is unlike the known results on the optimal TAS rules when the STx has instantaneous STx-PRx CSI. It is the optimal transmit power that depends on the interference constraint.

2) We derive a general expression for the average SEP that applies to any stochastic interference constraint and any number of antennas at the STx and SRx. We illustrate it for the average interference constraint and gain several insights.
3) Our numerical results show that the proposed rule achieves a several orders of lower SEP than on-off power adaptation and fixed power transmission considered in the literature. They also study how the interference constraint and multiple antennas at the STx and SRx impact the secondary and primary performances differently.

B. Outline and Notation

Section II presents the system model and formally states the optimization problem. The optimal TAS-CPA rule is derived and analyzed in Section III. Numerical results are presented in Section IV. Our conclusions follow in Section V.

Notation: Scalar variables are written in normal font and vector variables in bold font. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. $\mathbb{E}_X[\cdot]$ denotes expectation with respect to a random variable (RV) X. The indicator function is denoted by $I_{\{a\}}$, which is 1 if a is true and is 0 otherwise.

II. System Model and Problem Statement

The system model is shown in Figure 1. It consists of an STx and an SRx equipped with N_t and N_r antennas, respectively, and a PTx with a single antenna that communicates with a PRx equipped with N_p antennas. The STx is equipped with one RF chain, which is dynamically switched to one of the N_t antennas. The SRx employs selection combining (SC) [6]. For $n \in \{1,2,\ldots,N_r\}$ and $k \in \{1,2,\ldots,N_t\}$, h_{nk} denotes the instantaneous channel power gain from the k^{th} antenna of the STx to the n^{th} antenna of the SRx and for $i \in \{1,2,\ldots,N_p\}$, g_{ik} denotes the instantaneous channel power gain from the k^{th} antenna of the STx to the i^{th} antenna of the PRx. We consider Rayleigh fading. We assume that the STx-PRx channel gains are independent and identically distributed (i.i.d.) RVs [6], [8], [9]. Let $\mu_g = \mathbb{E}[g_{ik}]$.

A. Data Transmission and CSI Model

The STx selects an antenna $s \in \{1,2,\ldots,N_t\}$ and transmits a data symbol d with power P_s. The SRx receives a signal R_n at the n^{th} receive antenna. Let I_i denote the interference at the i^{th} antenna of the PRx due to secondary transmissions. Then, R_n and I_i are given by

$$R_n = \sqrt{P_s}\sqrt{h_{ns}}e^{j\theta_{ns}}d + N_n + W_n, \tag{1}$$

$$I_i = \sqrt{P_s}\sqrt{g_{is}}e^{j\varphi_{is}}d, \tag{2}$$

where $\mathbb{E}\left[|d|^2\right] = 1$, θ_{ns} and φ_{is} are the phases of the complex baseband channel gains of the STx-SRx and STx-PRx links, respectively, N_n is the additive white Gaussian noise, and W_n denotes the interference at the SRx due to primary transmissions. We assume $N_n + W_n$ is a circular symmetric complex Gaussian RV[1] with variance σ^2, which is the sum of thermal noise power and interference power [11], [13]. Let $h_k \triangleq \max_{1\le n\le N_r}\{h_{nk}\}$, $g_k \triangleq \sum_{i=1}^{N_p} g_{ik}$, $\mathbf{h} \triangleq [h_1,\ldots,h_{N_t}]$, and $\mathbf{g} \triangleq [g_1,\ldots,g_{N_t}]$.

CSI Model and Signaling Requirements: The STx knows only the statistics of the STx-PRx channel power gains, such as their average μ_g or their probability distribution [9], [12], [15]. It can acquire the statistics by listening to the signals transmitted by the PRx [6]. Note that unlike the acquisition of instantaneous CSI, this can take place over a time scale that is several orders of magnitude larger since the statistical CSI changes at a much slower rate [9].

The STx knows the instantaneous STx-SRx channel power gains $\mathbf{h}$, as is typical in conventional TAS [16]. It can estimate this from the pilots transmitted by the SRx, which is part of the same secondary system, and by exploiting reciprocity. The SRx performs coherent demodulation. For this, it only needs to know the complex channel gains from the selected transmit antenna s of the STx to itself. This can be acquired from a pilot transmitted by the STx along with the data [11].

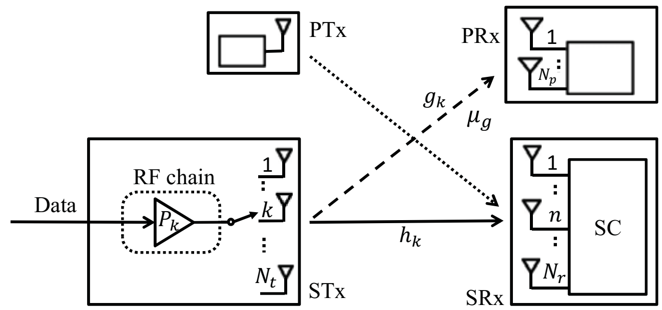

Fig. 1. System model that consists of an STx with N_t transmit antennas and one RF chain. It transmits data to an SRx with N_r antennas, which causes interference to a PRx with N_p antennas.

B. Stochastic Interference Constraint and Problem Statement

From (2), the total instantaneous interference power at all the antennas of the PRx is given by $P_s \sum_{i=1}^{N_p} g_{is} = P_s g_s$. A stochastic interference constraint is of the following form:

$$\mathbb{E}_{\mathbf{h},\mathbf{g}}\left[c\left(P_s, g_s\right)\right] \le G_t, \tag{3}$$

[1]This assumption is widely used due to its tractability. We refer the reader to [11] for a detailed discussion of the conditions under which it is applicable.

where $c(P_s, g_s)$ is the instantaneous interference penalty function that depends on the transmit power P_s and the STx-PRx channel power gain g_s, and G_t is a threshold. For example, for the average interference constraint [13], which limits $\mathbb{E}_{\mathbf{h},\mathbf{g}}[P_s g_s]$ to be below an interference power threshold τ, we have

$$c(P_s, g_s) = P_s g_s \text{ and } G_t = \tau. \tag{4}$$

A more general version of this is $c(P_s, g_s) = (P_s g_s)^m$ and $G_t = \tau^m$, where $m \geq 1$. We shall refer to m as the penalty exponent. For the interference-outage constraint [11], which requires the probability that $P_s g_s$ exceeds a threshold τ to be below $O_{\max}$, i.e., $\Pr(P_s g_s > \tau) \leq O_{\max}$, we have $c(P_s, g_s) = I_{\{P_s g_s > \tau\}}$ and $G_t = O_{\max}$.

A TAS-CPA rule ϕ maps $\mathbf{h}$ to an antenna s in the set $\{1, 2, \ldots, N_t\}$ and a transmit power P_s in the interval $[0, P_{\max}]$, where the peak power is motivated by practical power amplifier limitations. Thus, $(s, P_s) = \phi(\mathbf{h})$. Notice that due to the statistical CSI model, (s, P_s) does not depend on $\mathbf{g}$; it depends only on its statistics.

Our goal is to find the optimal TAS-CPA rule ϕ^* that minimizes the average SEP of the secondary system subject to a general stochastic interference constraint and the peak transmit power constraint. For this, the instantaneous SEP $\mathsf{S}(P_k, h_k)$ when the STx transmits with power P_k using antenna k is given by [17, (9.7)], [11]

$$\mathsf{S}(P_k, h_k) = c_1 \exp\left(-c_2 \frac{P_k h_k}{\sigma^2}\right), \text{ for } 1 \leq k \leq N_t, \tag{5}$$

where the constants c_1 and c_2 depend on the constellation [8], [9], [11].

Our problem can be mathematically written as:

$$\mathcal{P}: \min \quad \mathbb{E}_{\mathbf{h},\mathbf{g}}[\mathsf{S}(P_s, h_s)] \tag{6}$$

$$\text{s.t.} \quad \mathbb{E}_{\mathbf{h},\mathbf{g}}[c(P_s, g_s)] \leq G_t, \tag{7}$$

$$0 \leq P_s \leq P_{\max}, \tag{8}$$

$$(s, P_s) = \phi(\mathbf{h}). \tag{9}$$

Since s is a function of only $\mathbf{h}$, (7) can be recast as

$$\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_s)\right] \leq G_t, \tag{10}$$

where $\bar{\mathsf{C}}(P_s) = \mathbb{E}_{\mathbf{g}}[c(P_s, g_s)]$. For example, for the average interference constraint, $\bar{\mathsf{C}}(P_s) = \mathbb{E}_{\mathbf{g}}[P_s g_s] = P_s \mathbb{E}_{\mathbf{g}}[g_1] = P_s N_p \mu_g$. And, for the interference-outage constraint, $\bar{\mathsf{C}}(P_s) = \mathbb{E}_{\mathbf{g}}\left[I_{\{P_s g_s > \tau\}}\right] = F_g^c(\tau/P_s)$, where $F_g^c(\cdot)$ is the complementary cumulative distribution function (CDF) of the i.i.d. RVs $g_1, \ldots, g_{N_t}$.

III. General Solution: Optimal TAS-CPA Rule

To develop an SEP-optimal TAS-CPA rule, let us first consider an interference unconstrained system in which (10) is inactive. Since the SEP is a monotonically decreasing function of P_s and h_s, it is easy to see that the optimal rule is

$$s = \underset{k \in \{1,2,\ldots,N_t\}}{\arg\max} \{h_k\}, \tag{11}$$

$$P_s = P_{\max}. \tag{12}$$

Let $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{\max})\right] = \bar{\mathsf{C}}(P_{\max})$ denote the average interference penalty of this unconstrained rule. When $\bar{\mathsf{C}}(P_{\max}) \leq G_t$, which we shall refer to as the *unconstrained* regime, this rule satisfies the interference constraint in (10) and is optimal.

However, when $\bar{\mathsf{C}}(P_{\max}) > G_t$, which we shall refer to as the *constrained* regime, the unconstrained rule does not satisfy the interference constraint in (10). For this regime, define

$$\mathsf{SM}_k(p) \triangleq \mathsf{S}(p, h_k) + \lambda \bar{\mathsf{C}}(p), \quad \text{for } p \in [0, P_{\max}], \tag{13}$$

where λ is an interference penalty factor. We shall call $\mathsf{SM}_k(p)$ as the *selection metric* of antenna k. It is a function of the transmit power p. The solution of $\mathcal{P}$ is as follows.

Theorem *1:* The optimal antenna s^* and its transmit power P_{s^*} in the constrained regime are given by:

$$s^* = \underset{k \in \{1,2,\ldots,N_t\}}{\arg\max} \{h_k\}, \tag{14}$$

$$P_{s^*} = \underset{p \in [0, P_{\max}]}{\arg\min} \left\{\mathsf{S}(p, h_{s^*}) + \lambda \bar{\mathsf{C}}(p)\right\}, \tag{15}$$

where λ is set such that the interference constraint in (10) is met with equality, i.e., $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*})\right] = G_t$.

Proof: The proof is given in Appendix A. ■

The above result shows that for any stochastic interference constraint and with statistical CSI of the STx-PRx links, the optimal rule selects the antenna with the highest STx-PRx channel power gain and adjusts its transmit power to meet the constraint. While this result is intuitive since the STx does not know $\mathbf{g}$, it has not been proved in the literature for such a general setting to the best of our knowledge. Since it is conditioned on instantaneous values of STx-SRx channel gains, it holds even when these are correlated. It is unlike the TAS rules with instantaneous STx-PRx CSI [8], [10], [11], [13], in which the optimal antenna and its transmit power both depend on the interference constraint.

A. Optimal Transmit Power

We now derive the optimal transmit power P_{s^*} in closed-form. Due to space constraints, we focus on penalty functions of the form $c(P_s, g_s) = (P_s g_s)^m$, where $m \geq 1$. Consider first the average interference constraint $(m = 1)$.

Average Interference Constraint $(m = 1)$*:* Substituting (5) and $\bar{\mathsf{C}}(p) = p N_p \mu_g$ in (13) yields $\mathsf{SM}_{s^*}(p) = c_1 \exp\left(-c_2 p h_{s^*}/\sigma^2\right) + \lambda p N_p \mu_g$. Its minimum occurs at

$$p = \frac{\sigma^2}{c_2 h_{s^*}} \ln\left(\frac{h_{s^*}}{\eta}\right), \tag{16}$$

where $\eta = \lambda N_p \mu_g \sigma^2 / (c_1 c_2)$. As h_{s^*} increases, p initially increases and then decreases. It attains a maximum value of $c_1/(e\lambda N_p \mu_g)$ at $h_{s^*} = e\eta$. After taking into account the peak transmit power constraint in (8), the optimal transmit power P_{s^*} can be shown to take the following insightful form. When $P_{\max} \geq c_1/(e\lambda N_p \mu_g)$,

$$P_{s^*} = \begin{cases} 0, & \text{if } h_{s^*} \leq \eta, \\ \frac{\sigma^2}{c_2 h_{s^*}} \ln\left(\frac{h_{s^*}}{\eta}\right), & \text{else.} \end{cases} \tag{17}$$

Otherwise, for $P_{\max} < c_1/(e\lambda N_p \mu_g)$,

$$P_{s^*} = \begin{cases} 0, & \text{if } h_{s^*} \leq \eta, \\ \frac{\sigma^2}{c_2 h_{s^*}} \ln\left(\frac{h_{s^*}}{\eta}\right), & \text{if } \eta < h_{s^*} < h_{\min} \text{ or } h_{s^*} > h_{\max}, \\ P_{\max}, & \text{if } h_{\min} \leq h_{s^*} \leq h_{\max}, \end{cases} \tag{18}$$

where $h_{\min} \triangleq -\sigma^2/\left(c_2 P_{\max}\right) W_0\left(-c_2 P_{\max}\eta/\sigma^2\right)$, $h_{\max} \triangleq -\sigma^2/\left(c_2 P_{\max}\right) W_{-1}\left(-c_2 P_{\max}\eta/\sigma^2\right)$, and $W_l(\cdot)$ denotes the l^{th} branch of the Lambert-W function [18].

Generalized Average Interference Constraint ($m > 1$): Here, $\bar{\mathsf{C}}(p) = p^m \psi_m$, where $\psi_m = \mathbb{E}_{\mathbf{g}}\left[(g_1)^m\right]$ denotes the m^{th} moment of the STx-PRx channel power gain. Substituting this in (13) yields $\mathsf{SM}_{s^*}(p) = c_1 \exp\left(-c_2 p h_{s^*}/\sigma^2\right) + \lambda p^m \psi_m$. Its minimum occurs at

$$p = \frac{(m-1)\sigma^2}{c_2 h_{s^*}} W_0\left(\left[\frac{c_1 c_2 h_{s^*}}{\lambda m \psi_m \sigma^2}\right]^{\frac{1}{m-1}} \frac{c_2 h_{s^*}}{(m-1)\sigma^2}\right). \tag{19}$$

The optimal transmit power takes the above value if it is less than $P_{\max}$. Otherwise $P_{s^*} = P_{\max}$. Since the optimal power is computed explicitly, the optimal rule involves a comparison of only N_t quantities.

B. Performance Analysis

We now analyze the average SEP, denoted by ($\overline{\mathsf{SEP}}$), for any m. We then specialize and simplify it for $m = 1$. For tractability, we shall assume that $h_1, \ldots, h_{N_t}$ are i.i.d. Let $F_h(\cdot)$ and $f_h(\cdot)$ denote their CDF and probability density function. Let $\mu_h = \mathbb{E}[h_{nk}]$. Let $\Omega_s = P_{\max}\mu_h/\sigma^2$ denote the peak fading-averaged signal-to-interference-plus-noise ratio (SINR).

Result *1:* The average SEP of the secondary system when the STx selects antenna s in (14) is given by

$$\overline{\mathsf{SEP}} = N_t \int_0^\infty \left[F_h(h_1)\right]^{N_t-1} \mathsf{S}(P_1, h_1) f_h(h_1)\, dh_1, \tag{20}$$

where P_1 is the optimal transmit power when the STx selects antenna one, $F_h(x) = \left(1 - \exp(-x/\mu_h)\right)^{N_r}$, and $f_h(x) = N_r\left(1 - \exp(-x/\mu_h)\right)^{N_r-1} \exp(-x/\mu_h)/\mu_h$, for $x \geq 0$.

Proof: The proof is given in Appendix B. ■

The above expression applies to any number of transmit and receive antennas N_t and N_r. To understand it better, consider $m = 1$. As in Section III-A, two cases arise.

a) $P_{\max} \geq c_1/(e\lambda N_p\mu_g)$: As per (17), for $h_1 < \eta$, $P_1 = 0$ and $\mathsf{S}(0, h_1) = c_1$. Else, $P_1 = \sigma^2 \ln(h_1/\eta)/(c_2 h_1)$ and $\mathsf{S}(P_1, h_1) = \eta/h_1$. Substituting these in (20) and simplifying further, it can be shown that $\overline{\mathsf{SEP}} = T_1 + T_2$, where

$$T_1 = c_1\left(1 - \exp(-\eta/\mu_h)\right)^{N_t N_r}, \tag{21}$$

$$T_2 = \frac{N_t N_r c_1 \eta}{\mu_h} \sum_{k=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{k} (-1)^k \mathbf{E}_1\left(\frac{(k+1)\eta}{\mu_h}\right), \tag{22}$$

and $\mathbf{E}_1(\cdot)$ denotes the exponential integral [19, pp. xxxv]. Here, T_1 and T_2 correspond to the average SEP when the STx transmits with zero power and non-zero power, respectively. T_1 decreases as N_t or N_r increases. T_1 and T_2 increase as η increases. Also, they do not depend on $P_{\max}$.

b) $P_{\max} < c_1/(e\lambda N_p\mu_g)$: Substituting (18) in (20), it can be shown that $\overline{\mathsf{SEP}} = T_1 + \widehat{T}_2$, where T_1 is given in (21) and

$$\begin{aligned} \widehat{T}_2 = & N_t N_r c_1 \sum_{k=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{k} (-1)^k \\ & \times \left[\frac{e^{-(k+1+c_2\Omega_s)\frac{h_{\min}}{\mu_h}} - e^{-(k+1+c_2\Omega_s)\frac{h_{\max}}{\mu_h}}}{k+1+c_2\Omega_s}\right. \\ & + \frac{\eta}{\mu_h}\mathbf{E}_1\left(\frac{(k+1)\eta}{\mu_h}\right) - \frac{\eta}{\mu_h}\mathbf{E}_1\left(\frac{(k+1)h_{\min}}{\mu_h}\right) \\ & \left. + \frac{\eta}{\mu_h}\mathbf{E}_1\left(\frac{(k+1)h_{\max}}{\mu_h}\right)\right]. \end{aligned} \tag{23}$$

Here, $\widehat{T}_2$ decreases as $P_{\max}$ increases.

IV. Numerical Results and Benchmarking

We first benchmark the performance of the optimal TAS-CPA rule with the other rules that select the antenna as per (14) and adapt the transmit power as follows:

i) *Fixed Power [9], [12]:* The STx transmits with a fixed power $P_t \leq P_{\max}$, which is chosen such that the interference constraint is met with equality in the constrained regime.

ii) *On-Off Power Adaptation [6], [11]:* The STx transmits with power

$$P_s = \begin{cases} 0, & \text{if } h_s \leq \beta, \\ P_{\max}, & \text{else}, \end{cases} \tag{24}$$

where $\beta > 0$ is set such that the interference constraint is met with equality in the constrained regime.

For example, for the average interference constraint, $P_t = \min\{P_{\max}, \tau/(N_p\mu_g)\}$ and $\beta = -\mu_h \ln\left(1 - \left[1 - \tau/(P_{\max}N_p\mu_g)\right]^{\frac{1}{N_t N_r}}\right)$.

In our simulation setup, the PTx transmits with fixed power and the PRx employs maximal ratio combining. For this, we set $\mu_h = -114$ dB, $\mu_g = -125$ dB, $\sigma^2 = -109$ dBm, and the average channel power gain from the PTx to PRx as -103 dB. These values lead to a secondary peak fading-averaged SINR $\Omega_s = P_{\max}\mu_h/\sigma^2$ of 10 dB when $P_{\max}$ is 15 dBm and a primary average signal-to-noise ratio (SNR) of 21 dB when the PTx transmits with a fixed power of 15 dBm.[2]

Figure 2 plots the average SEP of the secondary system that is subject to the average interference constraint ($m = 1$) as a function of the interference power threshold τ. It compares the above power adaptation schemes for different values of N_t and N_r. i) For $\tau/\sigma^2 \leq P_{\max}N_p\mu_g/\sigma^2 = 0.9$ dB, the system is in the constrained regime regardless of the values of N_t and N_r. For both CPA and fixed power transmission, $\overline{\mathsf{SEP}}$

[2]This corresponds to a carrier frequency of 2.4 GHz, bandwidth of 1 MHz, 300 K temperature, and a noise figure of 5 dB. We consider the simplified path-loss model [17, Chap. 2.6] with the path-loss exponent of 3.7, a reference distance of 1 m, a distance of 100 m between the STx and SRx, a distance of 50 m between the PTx and PRx, and a distance of 200 m between the STx and PRx.

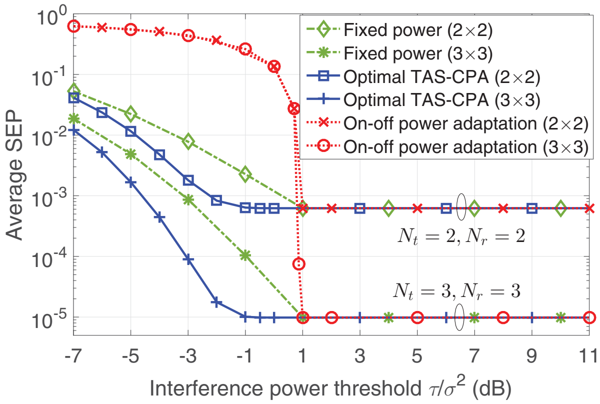

Fig. 2. Impact of power adaptation: Average SEP of the secondary system as a function of τ for different values of N_t and N_r ($m = 1$, $\Omega_s = 12$ dB, $N_p = 1$, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).

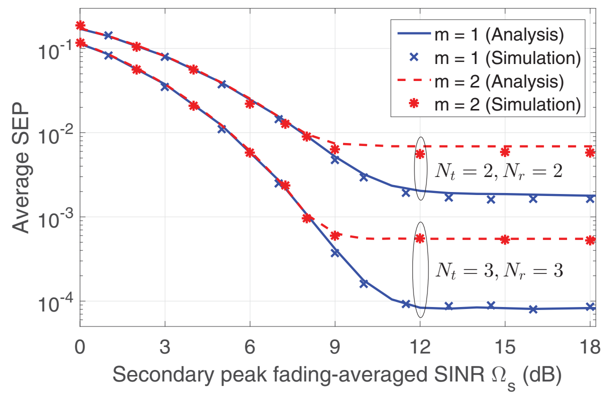

Fig. 3. Average SEP of the secondary system as a function of Ω_s for different values of N_t and N_r ($N_p = 2$, $\tau/\sigma^2 = 1$, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).

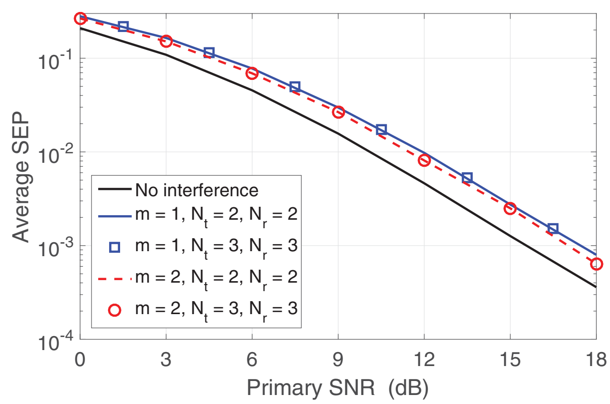

Fig. 4. Average SEP of the primary system as a function of its SNR for different values of N_t and N_r ($\Omega_s = 18$ dB, $N_p = 2$, $\tau/\sigma^2 = 1$, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).

decreases as τ increases. It also decreases significantly as N_t and N_r increases. However, the trends are different for on-off power adaptation. Here, $\overline{\text{SEP}}$ is insensitive to τ, N_t, and N_r. We observe a significant reduction in the average SEP due to CPA. For example, when $N_t = 3$, $N_r = 3$, and $\tau/\sigma^2 = -1$ dB, $\overline{\text{SEP}}$ of CPA is lower by a factor of 10 compared to fixed power transmission and 2 to 3 orders of magnitude lower compared to on-off power adaptation. On-off power adaptation does much worse because the STx transmits with zero power often in order to compensate for the interference it causes when it transmits with $P_{\max}$. ii) For $\tau/\sigma^2 > 0.9$ dB, the average SEPs of the different power adaptation techniques become the same and do not depend on τ as the system is in the unconstrained regime. In all of them the STx transmits with the peak power. Note that the SEP of the on-off power adaptation rule decreases rapidly as it enters the unconstrained regime. $\overline{\text{SEP}}$ decreases markedly as N_t or N_r increases.

Impact of Interference Constraint and Multiple Antennas: Figure 3 plots the average SEP of the secondary system, from simulations and analysis, as a function of its peak fading-averaged SINR Ω_s. Figure 4 plots the average SEP of the primary system, from simulations, as a function of its SNR. This is done for two values of the penalty exponent m. For small Ω_s, we see that the SEP of the secondary system does not depend on m because it is in the unconstrained regime. It decreases as Ω_s increases. The secondary system transitions to the constrained regime when $\Omega_s = \tau\mu_h/\left(N_p\mu_g\sigma^2\right)$ for $m = 1$ and $\Omega_s = \tau\mu_h/\left(\sqrt{N_p(N_p+1)}\mu_g\sigma^2\right)$ for $m = 2$. Now, for both constraints, the average SEP of the secondary system decreases and reaches an error floor. The error floor for $m = 1$ is significantly lower than that for $m = 2$, while the additional degradation in the SEP of the primary system is small. Thus, an interference constraint can have a very different impact on the primary and secondary systems, and its parameters need to be chosen judiciously. When N_t or N_r is increased, the average SEP of the primary system remains unchanged but that of the secondary decreases significantly. Lastly, we see that analysis and simulations match well in Figure 3.

V. Conclusions

We derived an optimal TAS-CPA rule for a peak-power constrained secondary system that had only statistical CSI of the STx-PRx links and was subject to a general, practically implementable stochastic interference constraint. We showed that it was the optimal transmit power that depended on the penalty function employed by the interference constraint. We derived the optimal transmit power in closed-form for the generalized average interference constraint. We also analyzed the average SEP of the optimal rule. We saw that continuous power adaptation markedly improved the secondary performance compared to both on-off power adaptation and fixed power transmission. While increasing the number of antennas improved the secondary performance without degrading the primary performance, increasing the penalty exponent m had a very different impact on the two systems.

Appendix

A. Proof of Theorem 1

Consider the TAS-CPA rule $(s^*, P_{s^*}) = \phi^*(\mathbf{h})$ defined by

$$(s^*, P_{s^*}) \triangleq \underset{k\in\{1,2,\ldots,N_t\},\ p\in[0,P_{\max}]}{\arg\min} \{\mathsf{SM}_k(p)\}, \quad (25)$$

where $\mathsf{SM}_k(p) = \mathsf{S}(p, h_k) + \lambda\bar{\mathsf{C}}(p)$ and $\lambda > 0$ is set such that the interference constraint is met with equality, i.e., $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*})\right] = G_t$.[3] Consider any other TAS-CPA rule $(s, P_s) = \phi(\mathbf{h})$ that satisfies the constraints (8) and (10). Clearly, by the construction of ϕ^*, it satisfies the constraints (8) and (10). Also, from (25), it follows that

$$\mathbb{E}\left[\mathsf{S}(P_{s^*}, h_{s^*}) + \lambda\bar{\mathsf{C}}(P_{s^*})\right] \leq \mathbb{E}\left[\mathsf{S}(P_s, h_s) + \lambda\bar{\mathsf{C}}(P_s)\right], \quad (26)$$

where the expectation is over $\mathbf{h}$. Rearranging terms yields

$$\mathbb{E}\left[\mathsf{S}(P_{s^*}, h_{s^*})\right] \leq \mathbb{E}\left[\mathsf{S}(P_s, h_s)\right] + \lambda\left(\mathbb{E}\left[\bar{\mathsf{C}}(P_s)\right] - \mathbb{E}\left[\bar{\mathsf{C}}(P_{s^*})\right]\right).$$

Since $\lambda > 0$ is chosen such that $\mathbb{E}\left[\bar{\mathsf{C}}(P_{s^*})\right] = G_t$ and $\mathbb{E}\left[\bar{\mathsf{C}}(P_s)\right] - G_t \leq 0$, we get $\mathbb{E}\left[\mathsf{S}(P_{s^*}, h_{s^*})\right] \leq \mathbb{E}\left[\mathsf{S}(P_s, h_s)\right]$. Thus, ϕ^* is SEP-optimal.

Consider two antennas l and q with channel power gains h_l and h_q, respectively, such that $h_l > h_q$. Let P_l and P_q minimize $\mathsf{SM}_l(p)$ and $\mathsf{SM}_q(p)$, respectively. As P_l minimizes the selection metric of antenna l, it follows that

$$\mathsf{S}(P_l, h_l) + \lambda\bar{\mathsf{C}}(P_l) \leq \mathsf{S}(P_q, h_l) + \lambda\bar{\mathsf{C}}(P_q), \quad (27)$$

Since the SEP decreases as the STx-SRx channel power gain increases, it follows that $\mathsf{S}(P_q, h_l) < \mathsf{S}(P_q, h_q)$. Combining this with (27), we get

$$\mathsf{S}(P_l, h_l) + \lambda\bar{\mathsf{C}}(P_l) < \mathsf{S}(P_q, h_q) + \lambda\bar{\mathsf{C}}(P_q), \quad (28)$$

which implies the $\mathsf{SM}_l(P_l) < \mathsf{SM}_q(P_q)$ if $h_l > h_q$. Thus, the antenna with the largest STx-SRx channel power gain is optimal. From (25), its transmit power is given by (15).

B. Proof of Result 1

Let $\Pr(\text{Err}|\mathbf{h})$ denote the probability of error conditioned on $\mathbf{h}$. Then, the average SEP is equal to $\overline{\mathsf{SEP}} = \mathbb{E}_{\mathbf{h}}\left[\Pr(\text{Err}|\mathbf{h})\right]$. Using the law of total probability and symmetry, we get

$$\overline{\mathsf{SEP}} = N_t \mathbb{E}_{\mathbf{h}}\left[\Pr(s = 1, \text{Err}|\mathbf{h})\right]. \quad (29)$$

And, $\Pr(s = 1, \text{Err}|\mathbf{h}) = \Pr(s = 1|\mathbf{h}) \Pr(\text{Err}|s = 1, \mathbf{h})$. Given $s = 1$ and $\mathbf{h}$, the probability of error equals $\mathsf{S}(P_1, h_1)$. Therefore, $\overline{\mathsf{SEP}} = N_t \mathbb{E}_{\mathbf{h}}\left[\Pr(s = 1|\mathbf{h})\, \mathsf{S}(P_1, h_1)\right]$. By the law of total expectation, we get

$$\overline{\mathsf{SEP}} = N_t \mathbb{E}_{h_1}\left[\Pr(s = 1|h_1)\, \mathsf{S}(P_1, h_1)\right]. \quad (30)$$

From (14), we know that antenna 1 is selected when $h_2 < h_1, \ldots, h_{N_t} < h_1$. Hence, $\Pr(s = 1|h_1) = \Pr(h_2 < h_1, \ldots, h_{N_t} < h_1|h_1)$. Conditioned on h_1, the events $h_2 < h_1, \ldots, h_{N_t} < h_1$ are mutually independent. Hence, we get

$$\Pr(s = 1|h_1) = \left[\Pr(h_2 < h_1|h_1)\right]^{N_t-1} = \left[F_h(h_1)\right]^{N_t-1}. \quad (31)$$

Substituting this in (30) and averaging over h_1 yields (20).

[3] The existence of λ can be shown for any $\bar{\mathsf{C}}(p)$ that is a convex function of p and has a continuous first derivative. It can also be shown for $\bar{\mathsf{C}}(p) = F_g^c(\tau/p)$, which arises for the interference outage constraint.

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