Optimal Relay and Antenna Selection in MIMO Cognitive Relay Network with Imperfect CSI

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***Abstract*—Cooperative relaying and multiple-input multiple-output (MIMO) transmission technologies exploit spatial diversity to improve the performance of the secondary users in an underlay cognitive radio network. We consider a MIMO cognitive relay network in which a secondary source and multiple relays have imperfect channel state information (CSI) of the interference links to the primary receiver. They sufficiently back-off their transmit powers on the basis of such CSI in order to adhere to an interference outage constraint. We propose an optimal relay and antenna selection scheme, which jointly selects a relay between the source and destination, a transmit antenna at the source, and a receive antenna at the destination to maximize the end-to-end signal-to-interference-plus-noise ratio (SINR) at the destination. To demonstrate the advantages of our proposed framework, we derive closed-form expression for the outage probability of the secondary network under non-identically distributed Rayleigh fading channels. We also derive an insightful expression for the asymptotic outage probability for high SINR and show that the diversity gain is lost when the interference power constraint is fixed. We then consider a practical scenario where the secondary users have only the mean channel power gains of the interference links. Under such CSI, we also derive an expression for the outage probability, and show that this can be used as a better performance/complexity tradeoff for high SINR.**

***Index Terms*—Cognitive radio network, MIMO, relay and antenna selection, outage probability, imperfect CSI.**

I. Introduction

Availability of sufficient spectrum is crucial for the success of new wireless technologies such as 5G and IEEE 802:11be [1], which target high data rates and massive connectivity using larger bandwidths. Regulatory authorities are, therefore, opening up pre-allocated spectrum bands for the shared and unlicensed use. Cognitive radio (CR) is a spectrum sharing technology that promises to significantly improve the utilization of scarce wireless spectrum. In the underlay mode of CR, a secondary user (SU) can simultaneously transmit on the same frequency band as a higher priority primary user (PU) so long as the interference it causes to the PU is tightly constrained. However, the interference constraint results in lower reliability and limited coverage for the SUs.

Cooperative relaying, which is also being considered in cellular systems [2], in combination with relay selection is an attractive, practicable solution to improve the performance of the SUs. In it, a single relay is selected to forward a message from a secondary source (S) to a secondary destination (D). The performance of the SUs can be further improved by using multiple-input multiple-output (MIMO) transmission technology. However, this requires enabling multiple transmit and receive radio frequency (RF) chains, which increases hardware complexity and cost. Antenna selection is a low complexity multiple-antenna technique that harnesses spatial diversity [3].

A fundamental difference that arises in underlay CR, when compared to conventional cooperative networks, is that the transmit powers of the SUs and the relay and antenna selection schemes depend on the interference caused to the PUs. The channel state information (CSI) available at the SUs about the interference links to the primary receiver and the nature of the interference constraint control this dependence. We summarize below the various models considered in the CR literature for antenna and/or relay selection.

A. Literature on Antenna and/or Relay Selection

With Perfect CSI of Interference Links: A single-relay cooperative MIMO network is considered in [3]–[5], and various schemes for transmit and/or receive antenna selection at S, D, and the relay are proposed. Instead, multiple spatially separated relays are considered in [6], [7]. In [6], instead of relay selection, the relays transmit simultaneously using beamforming. However, this increases feedback overhead and complexity to compute the beamforming matrices. In it, multiple antennas are considered only at D and the antenna with the highest signal-to-noise ratio (SNR) at D is selected. In [7], instead of antenna selection, beamforming is used at S and D, and the single-antenna relay that maximizes the signal-to-interference-plus-noise ratio (SINR) at D is selected.

With Imperfect CSI of Interference Links: Various transmit antenna selection schemes have been studied in [8], [9] when the SUs have imperfect CSI about the interference links to the primary receiver. However, they consider a single-relay cooperative MIMO network, which can not be readily extended to the multiple-relay network.

B. Novelty and Contributions

As can be seen from the aforementioned studies, there is no prior work which focuses on joint relay and antenna selection for multiple-relay cognitive MIMO network. Among multiple antenna techniques, transmit antenna selection, in which the transmitter selects one among multiple antennas and connects to the one available RF chain, is appealing as it exploits spatial diversity with low-hardware complexity. Furthermore, it needs only information of the channel power gains, which is easier to obtain compared to the complex baseband channel gains, and

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is robust to channel estimation errors [10]. Similarly, antenna selection at the receiver has the same advantages over maximal ratio combining, which needs multiple RF chains.

Motivated by the promising advantages of relay and transmit/receive antenna selection studied for the conventional cooperative networks [10] and to fill-in the gap, in this paper, we propose an optimal relay and antenna selection (ORAS) scheme for an underlay CR network that consists of a S, D, and multiple decode-and-forward (DF) relays, in which S and D are equipped with multiple antennas. This model can find applications in cellular communication systems and wireless sensor networks, since the base station and access point can be configured with multiple antennas while other nodes may be limited to a single antenna due to size, cost, and power constraints. Furthermore, we assume that the channel gains from/to different relays are non-identically distributed, and S and the relays have imperfect CSI about the interference links to the primary receiver due to channel estimation error.

In this paper, we make the following contributions: (i) We propose an ORAS scheme, which selects the optimal transmit antenna at S, the optimal receive antenna at D, and the optimal relay in order to maximize the end-to-end SINR at D, (ii) We derive an exact outage probability expression for the secondary network, in which S and the relays control their transmit powers as a function of the available CSI in order to satisfy an interference outage constraint and a peak transmit power constraint, and in which the corresponding ORAS scheme is employed, (iii) To gain more insights, we investigate the high SINR asymptotic regime and obtain the diversity order, and (iv) We also derive an outage probability expression for a special case when the secondary users have only the mean channel power gains of the interference links, and show that this can be used as a better performance/complexity tradeoff. We note that the analysis with imperfect CSI, which is useful for understanding the efficacy of CR in practical scenarios, is novel. Results for the special case with perfect CSI can be easily obtained from this, and are also novel.

Notations: The probability of an event A and the conditional probability of A given event B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. For a random variable (RV) X, $f_X(.)$ and $F_X(.)$ indicate the probability density function (PDF) and the cumulative distribution function (CDF), respectively. The indicator function $1_{\{a\}}$ is 1 if a is true and is 0 otherwise, $X \sim \mathcal{CN}(0, \sigma^2)$ means that X is a circularly symmetric, zero-mean, complex Gaussian RV with variance σ^2, and $X \sim \mathcal{E}\{\mu\}$ means that X is an exponential RV with mean μ.

II. System Model

The system model is shown in Fig. 1. We consider a primary network with a primary transmitter T that communicates to a primary receiver X. Both T and X are equipped with a single antenna. An underlay secondary network shares spectrum with this primary network. A secondary source S transmits data to a secondary destination D using L single-antenna DF relays $R_1, \ldots, R_L$. S and D are equipped with N_S and N_D antennas, respectively. Let S_j and D_k denote the j^{th} antenna of S and the k^{th} antenna of D, respectively, for $1 \leq j \leq N_S$ and $1 \leq k \leq N_D$.

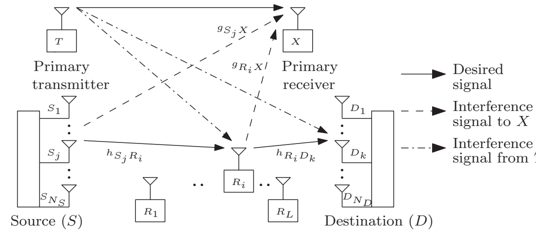

Fig. 1. Multiple-relay cognitive MIMO network with multiple antennas at the source and destination.

The complex baseband channel gain from S_j to R_i is $h_{S_jR_i}$, from R_i to D_k is $h_{R_iD_k}$, from S_j to X is g_{S_jX}, and from R_i to X is g_{R_iX}. We assume that the direct link between S and D is not available due to heavy shadowing or severe path loss. We consider Rayleigh fading and assume that the channel gains of the various links are mutually independent. The antennas at S are collocated, and so are the antennas at D. Hence, the channel gains from S to a specific relay are assumed to be identically distributed, and so are the channel gains from S to X and from a given relay to D. However, the channel gains $h_{S_jR_1}, \ldots, h_{S_jR_L}$ from S_j to different relays are non-identically distributed. So are the channel gains $h_{R_1D_k}, \ldots, h_{R_LD_k}$ and $g_{R_1X}, \ldots, g_{R_LX}$. This corresponds to a practical scenario where the relays are geographically separated apart. Therefore, $h_{S_jR_i} \sim \mathcal{CN}(0, \mu_{SR_i})$, $h_{R_iD_k} \sim \mathcal{CN}(0, \mu_{R_iD})$, $g_{S_jX} \sim \mathcal{CN}(0, \mu_{SX})$, and $g_{R_iX} \sim \mathcal{CN}(0, \mu_{R_iX})$, for $1 \leq i \leq L$, $1 \leq j \leq N_S$, and $1 \leq k \leq N_D$, where μ_{SR_i}, μ_{R_iD}, μ_{SX}, and μ_{R_iX} denote the respective mean channel power gains.

A. Data Transmission Protocol

S transmits data to D via a selected relay. A transmit antenna of S and a receive antenna of D are also selected. This selection happens prior to data transmission by S. If a relay R_i is selected, then in the first time slot, S transmits a data symbol by its selected antenna S_j with a transmit power P_{S_j}, and the selected relay R_i listens, where $j \in \{1, \ldots, N_S\}$ and $i \in \{1, \ldots, L\}$. In the second time slot, the DF relay R_i retransmits the decoded symbol with a transmit power P_{R_i}, and D receives it using the selected antenna D_k, where $k \in \{1, \ldots, N_D\}$.

The interferences at R_i and D due to primary transmissions are assumed to be Gaussian, as in [5], [6]. This assumption is justified even with one primary transmitter when it transmits a constant amplitude signal or transmits the orthogonal frequency division multiplexing signal [6]. It is justified with many primary transmitters by the central limit theorem. This is also valid when the interference seen at the relays and D is negligible [8]. Therefore, the instantaneous end-to-end SINR at D for DF relaying is defined as $\min\left\{\gamma_{S_jR_i}, \gamma_{R_iD_k}\right\}$ [5], where the SINR of the link between S_j and R_i is $\gamma_{S_jR_i} = P_{S_j}|h_{S_jR_i}|^2/\left(\sigma_0^2 + \sigma_{R_i}^2\right)$ and the SINR of the link between R_i and D_k is $\gamma_{R_iD_k} = P_{R_i}|h_{R_iD_k}|^2/\left(\sigma_0^2 + \sigma_D^2\right)$. Here, σ_0^2 is the variance of the Gaussian noise at the relays and D, and

$\sigma_{R_i}^2$ and σ_D^2 are the variances of the interference at R_i and D, respectively, due to primary transmissions.

B. CSI Model

A relay R_i is assumed to know the instantaneous channel power gains of $|h_{R_i D_k}|^2$, for $1 \leq k \leq N_D$, perfectly. It can estimate them by using a training protocol and exploiting channel reciprocity [6]. Similarly, S is assumed to know the instantaneous channel power gains of $|h_{S_j R_i}|^2$, for $1 \leq i \leq L$ and $1 \leq j \leq N_S$, perfectly.

In order to model noisy or imperfect CSI of the interference links, we adopt the following model [11], [12]. Let x_p denote the pilot symbol transmitted by the primary receiver X. Exploiting channel reciprocity, the signal received by the antenna S_j is given by $y_{S_j} = \sqrt{P_p} g_{S_j X} x_p + n_{S_j} + \alpha_{S_j}$, where P_p is the pilot transmit power, $|x_p|^2 = 1$, and $n_{S_j} \sim CN(0, \sigma_0^2)$ is the Gaussian noise at S_j. The interference at S_j due to transmissions by T is α_{S_j}. This is assumed to be Gaussian, as justified before. Therefore, $\alpha_{S_j} \sim CN(0, \sigma_S^2)$. Furthermore, $g_{S_j X} \sim CN(0, \mu_{SX})$ and is independent of n_{S_j} and α_{S_j}.

The minimum mean square error (MMSE) estimate $\hat{g}_{S_j X}$ for $g_{S_j X}$ is then given by $\hat{g}_{S_j X} = \frac{\sqrt{P_p} \mu_{SX} x_p^* y_{S_j}}{P_p \mu_{SX} + (\sigma_0^2 + \sigma_S^2)} = \rho_S g_{S_j X} + w_{S_j}$, where $\rho_S = \frac{P_p \mu_{SX}}{P_p \mu_{SX} + (\sigma_0^2 + \sigma_S^2)}$ and $w_{S_j} = \frac{\sqrt{P_p} \mu_{SX} x_p^* \left(n_{S_j} + \alpha_{S_j}\right)}{P_p \mu_{SX} + (\sigma_0^2 + \sigma_S^2)} \sim CN\left(0, \frac{P_p \mu_{SX}^2 (\sigma_0^2 + \sigma_S^2)}{\left(P_p \mu_{SX} + (\sigma_0^2 + \sigma_S^2)\right)^2}\right)$ is the channel estimation error [12]. Since w_{S_j} is independent of $g_{S_j X}$, it can be shown that $\hat{g}_{S_j X} \sim CN\left(0, \hat{\mu}_{SX}\right)$, where $\hat{\mu}_{SX} = \rho_S^2 \mathbb{E}\left[|g_{S_j X}|^2\right] + \mathbb{E}\left[|w_{S_j}|^2\right] = \rho_S \mu_{SX}$. Similarly, for relay R_i, the MMSE estimate $\hat{g}_{R_i X}$ for $g_{R_i X}$ satisfies $\hat{g}_{R_i X} \sim CN\left(0, \hat{\mu}_{R_i X}\right)$, where $\hat{\mu}_{R_i X} = \rho_i \mu_{R_i X}$ and $\rho_i = \frac{P_p \mu_{R_i X}}{P_p \mu_{R_i X} + (\sigma_0^2 + \sigma_{R_i}^2)}$.

C. Interference Outage Constraint and Power Control

It is not possible to meet a peak interference power constraint with imperfect CSI. Instead, we consider an interference outage constraint, which requires that the interference power at the primary receiver due to secondary transmissions cannot exceed a threshold I_{th} beyond a fraction of time p_{o} [12]. This is a generalization of the peak interference constraint, which corresponds to $p_{\text{o}} = 0$. Let $I_{S_j X} = P_{S_j} |g_{S_j X}|^2$ denote the instantaneous interference power at X due to transmissions by S_j. Then, the interference outage constraint is given by $\Pr\left(I_{S_j X} \leq I_{\text{th}}\right) \geq 1 - p_{\text{o}}$. Similarly, let $I_{R_i X} = P_{R_i} |g_{R_i X}|^2$ denote the instantaneous interference power at X due to transmissions by R_i. The corresponding interference outage constraint is given by $\Pr(I_{R_i X} \leq I_{\text{th}}) \geq 1 - p_{\text{o}}$.

We consider the following power control policy in which S_j sets its transmit power P_{S_j} as a function of the estimated channel power gain $|\hat{g}_{S_j X}|^2$ as $P_{S_j} = \min\left\{P_{\max}, \frac{I_{\text{th}}}{\tau_S |\hat{g}_{S_j X}|^2}\right\}$, where $P_{\max}$ is the peak transmit power of S and I_{th} is the peak interference threshold. Here, τ_S (≥ 1) is the source power back-off factor that is chosen in order to satisfy the interference outage constraint with equality [8]. The power back-off factors for all the antennas of S are identical because the channel gains from the different source antennas to relay R_i are assumed to be statistically identical, and so are the channel gains from the different source antennas to X. Similarly, the transmit power P_{R_i} of relay R_i is $P_{R_i} = \min\left\{P_{\max}, \frac{I_{\text{th}}}{\tau_i |\hat{g}_{R_i X}|^2}\right\}$, where τ_i (≥ 1) is the power back-off factor for relay R_i that is chosen in order to satisfy the interference outage constraint with equality.

D. ORAS Scheme

For the above transmit power control policy, we propose the optimal relay and antenna selection scheme that maximizes the end-to-end SINR of the secondary network. By it, the index i^* of the selected relay, the index j^* of the selected transmit antenna at S, and the index k^* of the selected receive antenna at D are jointly determined by

$$(i^*, j^*, k^*) = \underset{1 \leq i \leq L,\ 1 \leq j \leq N_S,\ 1 \leq k \leq N_D}{\arg\max} \min\left\{\gamma_{S_j R_i}, \gamma_{R_i D_k}\right\}. \tag{1}$$

The ORAS scheme can be implemented as follows:

- Step 1: S first computes the maximum SINR $U_i = \max\limits_{1 \leq j \leq N_S} \gamma_{S_j R_i}$ of the link between S and R_i, for $1 \leq i \leq L$. R_i computes the maximum SINR $V_i = \max\limits_{1 \leq k \leq N_D} \gamma_{R_i D_k}$ of the link between R_i and D, and feeds back to S, for $1 \leq i \leq L$.
- Step 2: S then computes the end-to-end maximum SINR $Z_i = \min\{U_i, V_i\}$, for $1 \leq i \leq L$. Using this, it selects the relay index $i^* = \underset{1 \leq i \leq L}{\arg\max} Z_i$ and the transmit antenna index $j^* = \underset{1 \leq j \leq N_S}{\arg\max} \gamma_{S_j R_{i^*}}$. It broadcasts the index i^* to all the relays, and R_{i^*} selects the receive antenna index $k^* = \underset{1 \leq k \leq N_D}{\arg\max} \gamma_{R_{i^*} D_k}$ of D.

Note that R_i needs to feedback only one value V_i instead of the SINRs $\gamma_{R_i D_k}$, for $1 \leq k \leq N_D$. This reduces the total amount of feedback required by R_i, for $1 \leq i \leq L$.

III. Outage Probability Analysis

We now analyze the outage probability of the secondary network for the ORAS scheme with imperfect CSI.

A. Computing Power Back-off Factors τ_S and τ_i

We first derive τ_S and τ_i in terms of the system parameters.

Lemma *1:* The probability $\mathfrak{I}_{S_j}$ that no interference outage occurs due to transmissions by any source antenna S_j, for $j \in \{1, \ldots, N_S\}$, is given by

$$\mathfrak{I}_{S_j} = 1 - e^{-\frac{\eta}{\mu_{SX}}} Q_1\left(\sqrt{\frac{2\eta}{\tau_S \rho_S \lambda_S}}, \sqrt{\frac{2\rho_S \eta}{\lambda_S}}\right) + \frac{1}{2}\left(1 + \frac{c_2}{c_3}\right) \times e^{-\frac{c_1 {c_4}^2}{2}} I_0\left(\sqrt{\tau_S} {c_4}^2\right) - \frac{c_2}{c_3} Q_1\left(c_4 \sqrt{\frac{c_1 - c_3}{2}}, c_4 \sqrt{\frac{c_1 + c_3}{2}}\right), \tag{2}$$

where $\eta = I_{\text{th}}/P_{\max}$, $\lambda_S = (1 - \rho_S)\mu_{SX}$, $c_1 = 1 + \tau_S + (1 - \rho_S)/\rho_S$, $c_2 = 1 - \tau_S + (1 - \rho_S)/\rho_S$, $c_3 = \sqrt{{c_1}^2 - 4\tau_S}$, $c_4 = \sqrt{2\eta/(\tau_S \lambda_S)}$, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind [13, (8.431.1)], and $Q_1(a, b)$ denotes the first-order Marcum Q-function [14, (4.34)]. The probability

$\mathfrak{I}_{R_i}$ that no interference outage occurs due to transmissions by relay R_i is the same as (2) except that τ_S, ρ_S, μ_{SX}, and λ_S are replaced by τ_i, ρ_i, μ_{R_iX}, and λ_i, respectively, where $\lambda_i = (1-\rho_i)\,\mu_{R_iX}$, and ρ_S and ρ_i are as defined in Section II-B.

Proof: The proof is relegated to Appendix A. ■

Since τ_S and τ_i are the solutions of the equations $\mathfrak{I}_{S_j} = 1 - p_o$ and $\mathfrak{I}_{R_i} = 1 - p_o$, respectively, they can be easily computed using routines such as `fsolve` in Matlab. We note that $\mathfrak{I}_{S_j}$ is identical for all $j \in \{1, \ldots, N_S\}$ but $\mathfrak{I}_{R_i}$ is different for all $i \in \{1, \ldots, L\}$. Equation (2) reveals the dependence of τ_S on the parameters P_p, p_o, μ_{SX}, and η. Similarly, τ_i depends on the parameters P_p, p_o, μ_{R_iX}, and η. Therefore, only statistical channel knowledge is required to determine τ_S and τ_i.

B. Exact Outage Probability Analysis

We now derive the outage probability given τ_S and τ_i, for $1 \le i \le L$. The end-to-end SINR γ_{e2e} for the ORAS scheme is given by $\gamma_{\text{e2e}} = \max_{1\le i\le L} Z_i$. The outage probability O_p is defined as $O_p = \Pr(\gamma_{\text{e2e}} \le \gamma_{\text{th}})$, where $\gamma_{\text{th}} = 2^{2r} - 1$ and r is the secondary target rate [8]. For notational simplicity, we define $\sigma_i^2 \triangleq \sigma_0^2 + \sigma_{R_i}^2$ and $\sigma^2 \triangleq \sigma_0^2 + \sigma_D^2$.

Result *1:* The outage probability O_p of the secondary network for the ORAS scheme under imperfect CSI is given by

$$O_p = \prod_{i=1}^{L} 1 - \left(1 - \left[1 - e^{-\frac{\sigma_i^2\gamma_{\text{th}}}{P_{\max}\mu_{SR_i}}} \times \left(1 - \frac{e^{-\frac{I_{\text{th}}}{P_{\max}\tau_S\rho_S\mu_{SX}}}}{1 + \frac{I_{\text{th}}\mu_{SR_i}}{\sigma_i^2\gamma_{\text{th}}\tau_S\rho_S\mu_{SX}}}\right)\right]^{N_S}\right) \times \left(1 - \left[\left(1 - e^{-\frac{\sigma^2\gamma_{\text{th}}}{P_{\max}\mu_{R_iD}}}\right)^{N_D}\left(1 - e^{-\frac{I_{\text{th}}}{P_{\max}\tau_i\rho_i\mu_{R_iX}}}\right) + \sum_{l=0}^{N_D}\binom{N_D}{l}(-1)^l \frac{e^{-\frac{I_{\text{th}}}{P_{\max}\tau_i}\left(\frac{1}{\rho_i\mu_{R_iX}} + \frac{\sigma^2\gamma_{\text{th}}l\tau_i}{I_{\text{th}}\mu_{R_iD}}\right)}}{1 + \frac{\sigma^2\gamma_{\text{th}}l\tau_i\rho_i\mu_{R_iX}}{I_{\text{th}}\mu_{R_iD}}}\right]\right). \quad (3)$$

Proof: The proof is relegated to Appendix B. ■

C. Asymptotic Outage Probability for High SINR

For notational simplicity, let $\sigma_i^2 = \sigma^2$, for $1 \le i \le L$. We define $\overline{\gamma}_P = P_{\max}/\sigma^2$ and $\overline{\gamma}_I = I_{\text{th}}/\sigma^2$. Let $\overline{\gamma}_P$ denote the secondary system SINR [5]. In order to gain more insights about system performance, we focus on $\overline{\gamma}_P \to \infty$, and consider the practical scenario where I_{th} is fixed and independent of $P_{\max}$, as considered in the literature [5], [9].

Corollary *1:* In the high SINR regime when $\overline{\gamma}_I$ is fixed and as $\overline{\gamma}_P \to \infty$, O_p in (3) can be shown to be

$$O_p \to \prod_{i=1}^{L} 1 - \left[1 - \left(\frac{\gamma_{\text{th}}\tau_S\rho_S\mu_{SX}}{\gamma_{\text{th}}\tau_S\rho_S\mu_{SX} + \overline{\gamma}_I\mu_{SR_i}}\right)^{N_S}\right] \times \left[1 - \sum_{l=0}^{N_D}\binom{N_D}{l}(-1)^l \frac{1 - \frac{\overline{\gamma}_I}{\overline{\gamma}_P\tau_i}\left(\frac{1}{\rho_i\mu_{R_iX}} + \frac{\gamma_{\text{th}}l\tau_i}{\overline{\gamma}_I\mu_{R_iD}}\right)}{1 + \frac{\gamma_{\text{th}}l\tau_i\rho_i\mu_{R_iX}}{\overline{\gamma}_I\mu_{R_iD}}}\right]. \quad (4)$$

It is worth noting from (4) that O_p saturates for higher values of $\overline{\gamma}_P$ and no diversity gain can be achieved due to the fixed interference power constraint. This result is valid for practical CR networks where X can only tolerate a limited amount of interference from S and relays. However, when the peak interference power I_{th} is proportional to the peak transmit power $P_{\max}$, full diversity order of $L\min\{N_S, N_D\}$ can be achieved. We skip this result due to space constraint.

D. Special Cases

1) Perfect CSI: In this case, S and the relays have perfect CSI of the interference links. Therefore, $\hat{g}_{S_jX} = g_{S_jX}$ and $\hat{g}_{R_iX} = g_{R_iX}$, for $1 \le j \le N_S$ and $1 \le i \le L$. This arises when $P_p \to \infty$, which corresponds to $\rho_S = \rho_i = 1$, for $1 \le i \le L$. Then, from the definitions of P_{S_j} and P_{R_i} in Section II-C, we note that the peak interference constraint at X due to the source and relay transmissions is always satisfied. Therefore, $p_o = 0$, $\tau_S = \tau_i = 1$. Substituting these values of τ_S, τ_i, ρ_S, and ρ_i in (3) and (4), we get the expressions for the exact and the asymptotic outage probability, respectively.

2) Mean value (MV)-based CSI: In this case, S and the relays have only the statistical knowledge of the interference links, which can considerably reduce the feedback burden as compared to obtaining the instantaneous CSI [8]. Specifically, S knows μ_{SX} and relay R_i knows μ_{R_iX}. The corresponding expressions for the outage probability can be shown to be

$$O_p = \prod_{i=1}^{L} 1 - \left[1 - \left(1 - e^{-\frac{\gamma_{\text{th}}\sigma_i^2}{P_{S_j}\mu_{SR_i}}}\right)^{N_S}\right] \times \left[1 - \left(1 - e^{-\frac{\gamma_{\text{th}}\sigma^2}{P_{R_i}\mu_{R_iD}}}\right)^{N_D}\right], \quad (5)$$

where $P_{S_j} = \min\left\{P_{\max}, \frac{I_{\text{th}}}{-\ln(p_o)\mu_{SX}}\right\}$ and $P_{R_i} = \min\left\{P_{\max}, \frac{I_{\text{th}}}{-\ln(p_o)\mu_{R_iX}}\right\}$, for $1 \le j \le N_S$ and $1 \le i \le L$.

IV. Numerical Results

In order to verify our analysis, we now present Monte Carlo simulation results. To generate the plots, we vary the secondary system SINR $\overline{\gamma}_P$. The average channel power gains of the various links are kept fixed, and are set to $\mu_{XY} = d_{XY}^{-\beta}$, where d_{XY} denotes the distance between the transmitting node X and the receiving node Y, and β denotes the path loss exponent. We set $\beta = 4$, the target rate $r = 1$ bps/Hz, and the 2D position of S, D, X, R_1, R_2, and R_3 as (0, 0), (1, 0), (1, 1), (1/4, 1/4), (1/2, 1/2), and (3/4, 3/4), respectively.

Fig. 2 plots O_p for two different values of $\overline{\gamma}_I$ and p_o. We see that for low values of $\overline{\gamma}_P$, as $\overline{\gamma}_P$ increases, the outage probability deceases and reaches a minimum value. This is because in this regime, the interference power constraint is inactive and the transmit powers of S and the selected relay become $P_{\max}$ more often. Therefore, as $P_{\max}$ increases, O_p decreases. For medium-to-high values of $\overline{\gamma}_P$, as $\overline{\gamma}_P$ increases, O_p increases. This is because in this regime, the interference power constraint becomes active and, consequently, the power

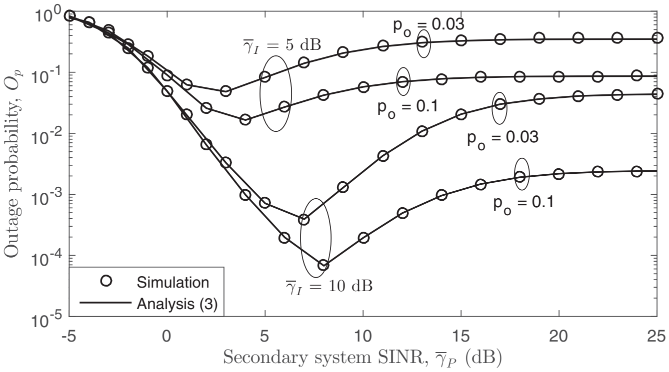

Fig. 2. Outage probability versus secondary system SINR for different values of $\overline{\gamma}_I$ and p_o ($P_p/\sigma^2 = 5$ dB, $N_S = 3$, $N_D = 4$, and $L = 3$).

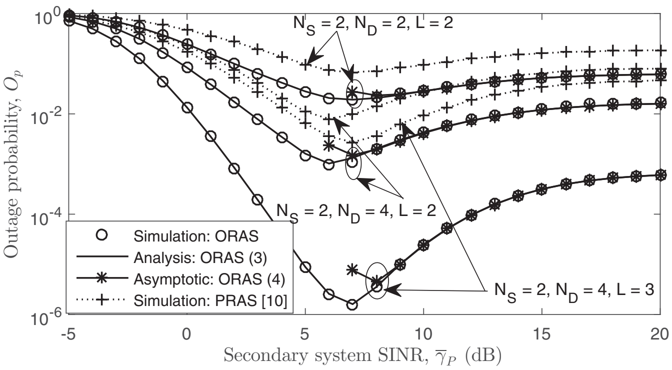

Fig. 3. Outage probability versus secondary system SINR for different number of antennas and relays ($P_p/\sigma^2 = 5$ dB, $p_o = 0.1$, $\overline{\gamma}_I = 8$ dB).

back off factors τ_S and τ_i increase from unity. This reduces the allowable transmit powers of S and the selected relay, which increases O_p. For sufficiently high values of $\overline{\gamma}_P$, the values of τ_S and τ_i become independent of $\overline{\gamma}_P$, and so are the transmit powers of S and the selected relay, which results O_p saturation. Note that for fixed $\overline{\gamma}_I$, as p_o increases, O_p decreases because the interference outage constraints become more relaxed. For fixed p_o, as $\overline{\gamma}_I$ increases, O_p decreases.

Fig. 3 plots O_p for three different values of N_S, N_D, and L. An excellent agreement between the analytical results and simulations is observed, and the asymptotic curves track the analytical results well. We observe similar trends as that of Fig 2. As expected, the outage probability decreases as the number of relays, or the number of antennas increase due to the increased spatial diversity. We also compare with the partial relay and antenna selection (PRAS) scheme proposed in [10]. It first selects a transmit antenna at S and a relay to maximize the SINR of the source-relay links, and then selects a receive antenna at D to maximize the SINR of the selected relay-destination links. In PRAS, V_i does not need to be fed back to S by R_i. However, it suffers from a significant loss in the outage probability compared to the proposed ORAS scheme.

Fig. 4 plots O_p under imperfect CSI for three values of P_p/σ^2. We see that as P_p/σ^2 increases, O_p decreases because the channel estimates become more perfect. For comparison, it also plots O_p under perfect CSI and MV-based CSI. As

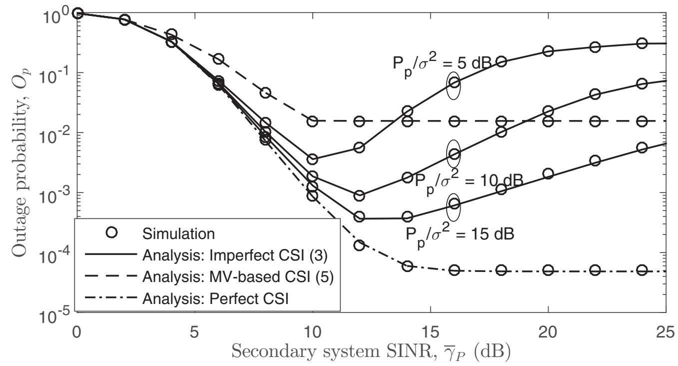

Fig. 4. Impact of different CSI availability ($r = 2$ bps/Hz, $p_o = 0.01$, $\overline{\gamma}_I = 15$ dB, $N_S = 4$, $N_D = 5$, and $L = 3$).

expected, for the entire range of $\overline{\gamma}_P$ considered, the minimum value of O_p is obtained with perfect CSI. In this case, as $\overline{\gamma}_P$ increases, O_p decreases for low-to-mid values of $\overline{\gamma}_P$, in which S and the selected relay set their transmit powers equal to $P_{\max}$ more often. For higher values of $\overline{\gamma}_P$, their transmit powers are limited by the peak interference power constraint, which limits O_p. The trends for MV-based CSI is similar to that of the perfect CSI. Unlike the case with imperfect CSI, τ_S and τ_i remain constant for a given p_o, which results O_p saturation for medium-to-high values of $\overline{\gamma}_P$. We see that it performs worst for low-to-mid values of $\overline{\gamma}_P$. However, for $\overline{\gamma}_P > 13.5$ dB, it outperforms the imperfect CSI case with $P_p/\sigma^2 = 5$ dB. This explains the advantage of the MV-based CSI as compared to the imperfect CSI for high SINRs as a better performance/complexity tradeoff.

V. Conclusions

We proposed an ORAS scheme for the multiple-relay cognitive MIMO network under imperfect CSI of the interference links. In it, the source and the selected relay sufficiently backed-off their transmit powers in order to meet the interference outage constraint. For this, we derived closed-form expressions for the exact and asymptotic outage probability. We saw that under the fixed interference power constraint, the outage probability saturated for higher values of SINR, which can be reduced by increasing either the number of antennas at S or D, or the number of relays. We also derived an expression for the outage probability when the SUs had only the mean channel power gains of the interference links, and showed that this can be used as a better performance/complexity tradeoff.

Appendix

A. Proof of Lemma 1

The probability $\mathfrak{I}_{S_j}$ that no interference outage occurs due to transmissions by S_j is given by

$$\mathfrak{I}_{S_j} = \Pr\left(\min\left\{P_{\max}, \frac{I_{\text{th}}}{\tau_S |\hat{g}_{S_j X}|^2}\right\} |g_{S_j X}|^2 \le I_{\text{th}}\right) = T_1 + T_2, \tag{6}$$

where $T_1 = \Pr\left(|g_{S_j X}|^2 \le \eta, |\hat{g}_{S_j X}|^2 \le \frac{\eta}{\tau_S}\right)$ and $T_2 = \Pr\left(|g_{S_j X}|^2 \le \tau_S |\hat{g}_{S_j X}|^2, |\hat{g}_{S_j X}|^2 > \frac{\eta}{\tau_S}\right)$. We now evaluate T_1 and T_2 separately.

1) Evaluating T_1: Using conditional CDF, T_1 can be rewritten as $T_1 = \frac{1}{\rho_S \mu_{SX}} \int\limits_0^{\eta/\tau_S} F_{|g_{S_j X}|^2 \,|\, |\hat{g}_{S_j X}|^2}(\eta|y)\, e^{-\frac{y}{\rho_S \mu_{SX}}}\, dy$. Note that $|g_{S_j X}|$ and $|\hat{g}_{S_j X}|$ are correlated RVs, and $|g_{S_j X}|^2 \sim \mathcal{E}\{\mu_{SX}\}$ and $|\hat{g}_{S_j X}|^2 \sim \mathcal{E}\{\rho_S \mu_{SX}\}$. Using the bivariate Rayleigh joint PDF of $|g_{S_j X}|$ and $|\hat{g}_{S_j X}|$ from [14, (6.2)], the conditional CDF $F_{|g_{S_j X}|^2 \,|\, |\hat{g}_{S_j X}|^2}(x|y)$ can be shown as

$$F_{|g_{Si}|^2 \,|\, |\hat{g}_{S_j X}|^2}(x|y) = 1 - Q_1\left(\sqrt{\frac{2y}{\lambda_S}}, \sqrt{\frac{2x}{\lambda_S}}\right), \quad (7)$$

where $\lambda_S = (1-\rho_S)\,\mu_{SX}$ and $Q_1(a,b)$ denotes the first-order Marcum Q-function [14, (4.34)].

Substituting (7) in the above expression for T_1, and using the variable transformation $\sqrt{y} = t$ and the identity in [15, (37)], we can show that

$$T_1 = 1 - e^{-\frac{\eta}{\tau_S \rho_S \mu_{SX}}} \left(1 - Q_1\left(\sqrt{\frac{2\eta}{\tau_S \lambda_S}}, \sqrt{\frac{2\eta}{\lambda_S}}\right)\right) - e^{-\frac{\eta}{\mu_{SX}}} Q_1\left(\sqrt{\frac{2\eta}{\tau_S \rho_S \lambda_S}}, \sqrt{\frac{2\rho_S \eta}{\lambda_S}}\right). \quad (8)$$

2) Evaluating T_2: Similarly, T_2 can be rewritten as $T_2 = \frac{1}{\rho_S \mu_{SX}} \int\limits_{\eta/\tau_S}^{\infty} \left(1 - Q_1\left(\sqrt{\frac{2y}{\lambda_S}}, \sqrt{\frac{2\tau_S y}{\lambda_S}}\right)\right) e^{-\frac{y}{\rho_S \mu_{SX}}}\, dy$. Using the variable transformation $\sqrt{y} = t$ and the identity in [15, (55)], we can show that

$$T_2 = e^{-\frac{\eta}{\tau_S \rho_S \mu_{SX}}} \left(1 - Q_1(c_4, \sqrt{\tau_S} c_4)\right) + \frac{1}{2}\left(1 + \frac{c_2}{c_3}\right) e^{-\frac{c_1 {c_4}^2}{2}} \times I_0\left(\sqrt{\tau_S} {c_4}^2\right) - \frac{c_2}{c_3} Q_1\left(c_4\sqrt{\frac{c_1 - c_3}{2}}, c_4\sqrt{\frac{c_1 + c_3}{2}}\right), \quad (9)$$

where c_1, c_2, c_3, and c_4 are defined in the Lemma statement. Substituting (8) and (9) in (6) and simplifying further yields (2).

The derivation for $\Im_{R_i}$ is along similar lines, and is skipped.

B. Proof of Result 1

Since $Z_1, \ldots, Z_L$ are independent RVs and U_i is independent of V_i, O_p simplifies to

$$O_p = \prod_{i=1}^{L} F_{Z_i}(\gamma_{\text{th}}) = \prod_{i=1}^{L} 1 - \left(1 - F_{U_i}(\gamma_{\text{th}})\right)\left(1 - F_{V_i}(\gamma_{\text{th}})\right). \quad (10)$$

1) Evaluating $F_{U_i}(\gamma_{th})$: Since $\gamma_{S_1 R_i}, \ldots, \gamma_{S_{N_S} R_i}$ are i.i.d. RVs, $F_{U_i}(\gamma_{\text{th}}) = \left(F_{\gamma_{S_j R_i}}(\gamma_{\text{th}})\right)^{N_S}$. The CDF of $\gamma_{S_j R_i}$ can be written as

$$F_{\gamma_{S_j R_i}}(\gamma_{\text{th}}) = \frac{1}{\mu_{SR_i}} \int\limits_0^{\infty} F_{P_{S_j}}\left(\frac{\sigma_i^2 \gamma_{\text{th}}}{\alpha}\right) e^{-\frac{\alpha}{\mu_{SR_i}}}\, d\alpha, \quad (11)$$

where the CDF of P_{S_j} can be shown to be

$$F_{P_{S_j}}(x) = \begin{cases} e^{-\frac{I_{\text{th}}}{x \tau_S \rho_S \mu_{SX}}}, & x \le P_{\max}, \\ 1, & x > P_{\max}. \end{cases} \quad (12)$$

Substituting (12) in (11) and simplifying further, we get the CDF of U_i by the term $[.]^{N_S}$ in (3).

2) Evaluating $F_{V_i}(\gamma_{th})$: From the definition of P_{R_i} in Section II-C, we note that conditioned on $|\hat{g}_{R_i X}|^2$, the RVs $\gamma_{R_i D_1}, \ldots, \gamma_{R_i D_{N_D}}$ are i.i.d. Therefore, the CDF of V_i is

$$F_{V_i}(\gamma_{\text{th}}) = \int\limits_0^{\infty} \Pr\left(V_i \le \gamma_{\text{th}} \,\Big|\, |\hat{g}_{R_i X}|^2 = y\right) \frac{e^{-\frac{y}{\rho_i \mu_{R_i X}}}}{\rho_i \mu_{R_i X}}\, dy. \quad (13)$$

Using the law of total probability, the conditional probability term in (13) can be written as $\Pr\left(V_i \le \gamma_{\text{th}} \,\Big|\, |\hat{g}_{R_i X}|^2 = y\right) = \left(1 - e^{-\frac{\sigma^2 \gamma_{\text{th}}}{P_{\max} \mu_{R_i D}}}\right)^{N_D} 1_{\left\{y \le \frac{I_{\text{th}}}{\tau_i P_{\max}}\right\}} + \left(1 - e^{-\frac{\sigma^2 \gamma_{\text{th}} \tau_i y}{I_{\text{th}} \mu_{R_i D}}}\right)^{N_D} 1_{\left\{y > \frac{I_{\text{th}}}{\tau_i P_{\max}}\right\}}$. Substituting this in (13) and by using binomial expansion, we get the CDF of V_i by the second $[.]$ term in (3).

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