Low-Complexity Joint Antenna Selection and Beamforming for an IRS Assisted System

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2021 IEEE Wireless Communications and Networking Conference (WCNC) | 978-1-7281-9505-6/20/$31.00 ©2021 IEEE | DOI: 10.1109/WCNC49053.2021.9417367

***Abstract*—Intelligent reflecting surface (IRS), which uses passive reflective elements instead of active radio frequency (RF) chains, is a cost and energy-efficient solution to improve the wireless system performance. With a similar objective, transmit antenna selection (AS) reduces the number of RF chains at the base station while harnessing the benefits of multiple antennas. In our work, we focus on joint optimization of antenna subset and transmit beamforming at the base station, and passive beamforming at the IRS to maximize the receive signal power. For single AS, we first derive a closed-form optimal rule. We then propose a simpler AS rule, which significantly reduces the computational complexity and the number of pilot transmissions required. For a system with N_t antennas at the base station and N IRS elements, the optimal AS rule requires $N_t + N_tN$ pilots. However, the proposed simpler rule requires only $2N_t+N$ pilots. For subset AS, we develop a manifold optimization based algorithm. To reduce its subset search complexity, which is exponential in the number of RF chains at the base station, we propose an alternating optimization based iterative algorithm. Our numerical results show that the proposed simpler AS rules are near optimal.**

***Index Terms*—Intelligent reflecting surface, antenna selection, beamforming, manifold optimization, alternating optimization.**

I. Introduction

Intelligent reflecting surface (IRS) is being envisioned as a key technology for the sixth generation (6G) wireless communication systems to achieve a smart radio environment [1], [2]. The current 5G technologies that are based on large antenna arrays and the use of high frequencies would need a large number of expensive radio frequency (RF) chains. These RF chains consist of amplifiers, mixers, filters, and signal converters [3]. Instead, IRS consists of low-cost passive reflective elements such as printed dipoles [4], [5]. Hence, it improves energy and cost-efficiency. Each passive reflecting element of the IRS is capable of inducing a known phase shift, which can be programmed by a controller, to the incident electromagnetic wave. It enables passive beamforming to improve the receive signal power.

Similar to IRS, transmit antenna selection (AS) is a technology that improves energy and cost-efficiency by reducing the number of RF chains at the transmitter. In it, a transmitter selects a subset of antennas and connects them to the available RF chains, which are smaller in number than the antenna elements. AS achieves full diversity with fewer RF chains. It is part of wireless standards such as Long-Term Evolution and 802.11n [3]. In our work, we focus on AS at the base station (BS) assisted by passive beamforming at the IRS to improve the performance with low hardware cost.

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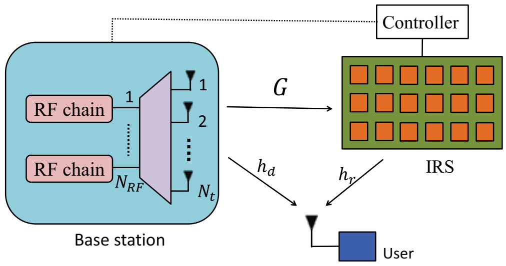

Fig. 1. System model that consists of a base station, which is equipped with N_t antennas and N_{RF} RF chains, communicating to a single antenna user with the help of an N element IRS.

Joint transmit beamforming at the BS and the passive beamforming at the IRS is studied extensively in the literature [4]–[8]. To minimize the transmit power at the BS, a local optimal solution is developed in [8] using an alternating optimization technique. Different optimization techniques are studied in [4]–[7] to maximize the receive signal power. A fixed point iteration method is proposed in [7] and a semi-definite relaxation (SDR) based technique, which yields an approximate solution, is proposed in [6]. A conjugate-gradient-based manifold optimization technique, which improves performance compared to SDR based technique, is developed in [4]. It converges to a local optimum solution. Furthermore, a branch-and-bound (BnB) algorithm, which converges to the global optimal solution, is presented in [5]. It is also shown that the performance of the manifold optimization based algorithm is near optimal with lower complexity. Most of these works assume that the channel state information (CSI) is available at both the BS and IRS, which is practically challenging due to the passive nature of the IRS elements. Furthermore, they focus on the transmit beamforming at the BS, which needs number of RF chains equal to the number of antennas. To the best of our knowledge, jointly optimal AS at the BS and passive beamforming at the IRS is not studied in the literature. However, a sub-optimal AS rule that selects a single antenna with the highest channel power gain from the BS to IRS is considered in [9].

A. Focus and Contributions

We focus on an IRS assisted communication system with a multiple antenna base station that employs AS and communicates to a single antenna user. Our objective is to develop

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a jointly optimal subset antenna selection, transmit beamforming at the BS, and passive beamforming at the IRS to maximize the receive signal power. Our problem formulation is novel and practical in the following aspects. Firstly, joint AS and beamforming is not studied for an IRS assisted system in the literature and the optimal AS rule is not known. Secondly, unlike [4]–[6], we do not assume any CSI at the IRS. Thirdly, our AS rule design reduces the number of pilots required.

Contributions

i) Single AS: We derive an optimal AS rule that selects the jointly optimal antenna at the BS and optimal reflection coefficient of each IRS element. We show that the optimal antenna and reflection coefficient are decoupled in nature. We then propose a simpler yet near-optimal AS rule that significantly reduces the number of pilot transmissions.

ii) Subset AS; We develop a manifold optimization based algorithm that jointly finds an antenna subset, transmit beamforming at the BS, and passive beamforming at the IRS. It searches over all possible subsets to find the optimal subset. To reduce this search complexity, we propose a subset AS algorithm that uses the alternating optimization technique.

iii) Numerical Results: Our results show that for a given number of RF chains at the BS, an increase in the number of antennas at the BS or the passive elements at the IRS improves system performance significantly. They also show that the proposed low-complexity AS rules are near optimal.

Outline: Section II presents our system model and problem statement. AS rules are developed in Section III. Numerical results are presented in Section IV. Our conclusions follow in Section V.

Notations: Scalars are denoted by lower-case letters. Vectors and matrices are denoted by boldface lower-case and capital letters, respectively. $\mathbb{C}^{m\times n}$ denotes the set of all complex-valued matrices of size $m \times n$ and $j = \sqrt{-1}$. $|a|$, $\arg(a)$, and a^* denote the absolute value, phase, and conjugate, respectively, of a complex number a. $\|\mathbf{x}\|$, $\mathbf{x}^\dagger$, and $[\mathbf{x}]_n$ denote the 2-norm, conjugate transpose, and n^{th} element of vector $\mathbf{x}$.

II. System Model

Our system model is shown in Figure 1. It consists of a BS that is equipped with N_t antennas and $N_{RF} \le N_t$ number of RF chains. It communicates to a single antenna user with the help of an IRS equipped with N passive reflective elements and a controller. The BS dynamically selects N_{RF} antennas from the set of antennas $\{1, 2, \ldots, N_t\}$, connects them to the RF chains available, and performs transmit beamforming. The IRS controller receives the reflection coefficients from the BS through a dedicated control link. It programs the reflectors to perform passive beamforming. We assume a quasi-static flat-fading channel model [4], [6]. Furthermore, we consider a time-division duplexing (TDD) mode of operation to exploit reciprocity and reduce CSI feedback overhead.

Let $\mathbf{h}_r^\dagger = \left[h_{r,n}^*\right] \in \mathbb{C}^{1\times N}$ denote the complex baseband channel gain vector from the IRS to the user. Let $\mathbf{h}_d^\dagger = \left[h_{d,k}^*\right] \in \mathbb{C}^{1\times N_t}$ and $\mathbf{G} = [g_{nk}] \in \mathbb{C}^{N\times N_t}$ denote the complex channel gain vector from the BS to the user and complex channel gain matrix from the BS to the IRS, respectively. Let $\mathcal{S}$ denote the set of all possible subsets of set $\{1, 2, \ldots, N_t\}$ each containing N_{RF} elements. Let $S \in \mathcal{S}$ denote a subset containing indices of N_{RF} antennas selected. Let $\mathbf{h}_{d,S}^\dagger \in \mathbb{C}^{1\times N_{RF}}$ denote the channel gain vector from the BS to the user corresponding to the subset S and $\mathbf{G}_S \in \mathbb{C}^{N\times N_{RF}}$ denote the sub-matrix that contains columns of $\mathbf{G}$ corresponding to the antenna indices in S. The reflection coefficient of n^{th} IRS element is denoted by $x_n = \beta e^{j\theta_n}$, where $\theta_n \in [0, 2\pi]$ and $\beta \in [0, 1]$ are its phase shift and reflection loss. Let $\mathbf{x} = [x_1, \ldots, x_N]^\dagger$ be the passive beamforming vector at the IRS and $\mathbf{w} = [w_k] \in \mathbb{C}^{N_{RF}\times 1}$ denote the transmit beamforming vector at the BS.

The BS transmits data symbol d using a subset of antennas in S. Then the user receives a signal $\mathbf{h}_{d,S}^\dagger \mathbf{w} d$ through the direct link (BS $\rightarrow$ user). The signal transmitted from antenna k and reflected through the n^{th} IRS element, which applies a reflection coefficient x_n, observes a cascaded channel $h_{r,n}^* x_n g_{nk}$. Hence, the user receives $h_{r,n}^* x_n g_{nk} w_k d$ through the reflected link (BS $\rightarrow$ IRS $\rightarrow$ user). The composite signal received through all the selected antennas and all the IRS elements is given by $\sum_{n=1}^{N} h_{r,n}^* x_n \left[\mathbf{G}_S \mathbf{w}\right]_n d$, where $\left[\mathbf{G}_S \mathbf{w}\right]_n$ denotes the n^{th} element of the vector $\mathbf{G}_S\mathbf{w}$. It can be written as $\mathbf{x}^\dagger \operatorname{diag}\left(\mathbf{h}_r^\dagger\right) \mathbf{G}_S \mathbf{w} d$, where $\operatorname{diag}\left(\mathbf{h}_r^\dagger\right)$ denote the diagonal matrix with elements of $\mathbf{h}_r^\dagger$ as its diagonal elements. Therefore, the signal y received at the user through the direct and reflected links is given by

$$y = (\mathbf{h}_{d,S}^\dagger + \mathbf{x}^\dagger \mathbf{H}_r \mathbf{G}_S)\mathbf{w} d + z, \tag{1}$$

where $\mathbf{H}_r = \operatorname{diag}\left(\mathbf{h}_r^\dagger\right)$ and z denote the additive white Gaussian noise at the user with zero mean and variance σ^2.

CSI Assumptions and Acquisition Procedure [10], [11]: We assume that the BS knows direct link channel gain vector $\mathbf{h}_d^\dagger$ and cascaded channel gain matrix $\mathbf{H}_r\mathbf{G}$ of the reflected link. The user sends pilot symbols and the BS estimates these channel gains in a two-phase method. In the first phase, the IRS is turned off and the BS estimates direct link channel gains. In the second phase, the IRS is turned on and the BS estimates the sum of the direct link and reflected link channel gain. This can be done by either turning on only one IRS element at a time [10] or turning all of them on and using rows of a discrete Fourier transform (DFT) matrix as passive beamforming vectors [11]. Individual channel gains of the BS to IRS and the IRS to user links, which are difficult to obtain due to the passive nature of the IRS, are not needed at the BS. Furthermore, no CSI is assumed at the IRS. The BS computes the passive beamforming vector $\mathbf{x}$ based on this CSI acquired and communicates to the IRS controller through a control link.

A. Problem Statement

We now state our problem formally. From (1) the instantaneous signal-to-noise ratio (SNR) at the receiver, when the BS transmits using subset S with transmit beamforming vector $\mathbf{w}$

and the IRS employs a passive beamforming vector $\mathbf{x}$ is given by

$$\mathsf{SNR}\left(S, \mathbf{w}, \mathbf{x}\right) = \left|(\mathbf{h}_{d,S}^{\dagger} + \mathbf{x}^{\dagger}\mathbf{H}_r\mathbf{G}_S)\mathbf{w}\right|^2 / \sigma^2. \quad (2)$$

Let $\mathsf{R}\left(S, \mathbf{w}, \mathbf{x}\right)$ denote the instantaneous rate. It is given by

$$\mathsf{R}\left(S, \mathbf{w}, \mathbf{x}\right) = \log_2\left(1 + \mathsf{SNR}\left(S, \mathbf{w}, \mathbf{x}\right)\right). \quad (3)$$

Similarly, the symbol error probability (SEP), which we denote by $\mathsf{SEP}\left(S, \mathbf{w}, \mathbf{x}\right)$, is given by [12, eq. (14)]

$$\mathsf{SEP}\left(S, \mathbf{w}, \mathbf{x}\right) = c_1 \exp\left(-c_2 \mathsf{SNR}\left(S, \mathbf{w}, \mathbf{x}\right)\right), \quad (4)$$

where c_1 and c_2 are modulation specific constants. From above, we see that maximizing instantaneous signal power maximizes rate and minimizes SEP. Thus, our objective is to maximize signal power at the receiver.

Constraints: The BS is subject to peak transmit power constraint. It limits the total instantaneous transmit power from the N_{RF} antennas selected to be below the maximum total transmit power $P_{\max}$ allowed, i.e., $\|\mathbf{w}\|^2 \leq P_{\max}$. We set $\beta = 1$ as our goal is to maximize the signal power. Hence, the modulus of each reflection coefficient at the IRS should be one, i.e., $|x_n| = 1, \ \forall n$, which in general is referred to as unit modulus constraint.

Optimization Problem: Our goal is to jointly solve for a subset of antennas S, transmit beamforming vector $\mathbf{w}$ at the BS, and passive beamforming vector $\mathbf{x}$ at the IRS to maximize receive signal power subject to peak transmit power constraint at the BS and unit modulus constraint of IRS reflection coefficients. The optimization is over a space of discrete sets of size N_{RF}, complex vectors $\left\{\mathbf{w} \in \mathbb{C}^{N_{RF} \times 1} : \|\mathbf{w}\|^2 \leq P_{\max}\right\}$, and $\left\{\mathbf{x} \in \mathbb{C}^{N \times 1} : |x_1| = 1, \ldots, |x_N| = 1\right\}$. Optimization problem can be written as

$$\mathcal{P}: \quad \max_{S, \mathbf{w}, \mathbf{x}} \quad \left|(\mathbf{h}_{d,S}^{\dagger} + \mathbf{x}^{\dagger}\mathbf{H}_r\mathbf{G}_S)\mathbf{w}\right|^2, \quad (5)$$

$$\text{s.t.} \quad \|\mathbf{w}\|^2 \leq P_{\max}, \quad (6)$$

$$|x_n| = 1, \ \forall \ n = 1, \ldots, N. \quad (7)$$

The above problem $\mathcal{P}$ is non-convex as the objective function is non-concave and unit-modulus constraint is non-convex. To the best of our knowledge, there is no simple tractable solution to this problem. In the next section, we shall propose low complexity yet near-optimal solutions for $\mathcal{P}$.

III. Antenna Selection With IRS

In this section, we first derive the optimal solution of $\mathcal{P}$ for a single antenna selection ($N_{RF} = 1$) scenario. We then develop a manifold optimization based algorithm for subset antenna selection ($N_{RF} > 1$). For both scenarios, we also propose AS rules that significantly reduce the computational complexity and number of pilot transmissions.

A. Single Antenna Selection ($N_{RF} = 1$)

Let $s \in \{1, 2, \ldots, N_t\}$ denote the index of the antenna selected. Here, the received signal y in (1) reduces to

$$y = \left(h_{d,s}^* + \sum_{n=1}^{N} h_{r,n}^* g_{ns} x_n\right) w_s d + z. \quad (8)$$

1) Optimal AS rule: We first present the optimal AS rule.

Result *1:* For an IRS assisted communication system with single antenna selection at the BS, the optimal antenna s_{opt}, its transmit power $w_{s_{\text{opt}}}$, and optimal passive reflection coefficient $x_{n,\text{opt}}$ are given by

$$s_{\text{opt}} = \underset{k \in \{1,2,\ldots,N_t\}}{\arg\max} \left\{|h_{d,k}| + \sum_{n=1}^{N} \left|h_{r,n}^* g_{nk}\right|\right\}, \quad (9)$$

$$x_{n,\text{opt}} = \exp\left(j\left[\arg\left(h_{d,s_{\text{opt}}}^*\right) - \arg\left(h_{r,n}^* g_{n s_{\text{opt}}}\right)\right]\right), \ \forall \ n, \quad (10)$$

$$w_{s_{\text{opt}}} = \sqrt{P_{\max}}. \quad (11)$$

Proof: The proof is given in Appendix A. ■

Insights: The selection metric of each antenna is the sum of the absolute values of the direct link and the N reflected link channel gains. The optimal antenna is the one with the highest selection metric. The optimal reflection coefficient of each IRS element is the difference between the phase of the direct link and the reflected link. Optimal antenna depends only on the absolute values of the channel gains, whereas the optimal reflection coefficient depends only on their phases. We see a decoupled structure between them.

Number of Pilot Transmissions Required and Computational Complexity: Here, the BS has only one RF chain, which it switches to each antenna, to estimate the channel gains. Hence, the user needs to transmit N_t pilots in the first phase to estimate direct link channel gains and $N_t N$ pilots in the second phase to estimate reflected link channel gains. In total, we need $N_t + N_t N$ pilots. Furthermore, we need $\mathcal{O}(N_t N)$ computations to select the optimal antenna and its reflection coefficients.

2) Low-Complexity AS (LAS) Rule: We now propose an AS rule that reduces the number of pilot transmissions and the computations required. It selects an antenna s that maximizes the following selection metric $|h_{d,k}| + \left|\sum_{n=1}^{N} h_{r,n}^* g_{nk}\right|$, which lower bounds the optimal selection metric of antenna k given in (9). Then, the reflection coefficient is computed as per (10) for the antenna selected. Hence, LAS rule is given by

$$s = \underset{k \in \{1,2,\ldots,N_t\}}{\arg\max} \left\{|h_{d,k}| + \left|\sum_{n=1}^{N} h_{r,n}^* g_{nk}\right|\right\}, \quad (12)$$

$$x_n = \exp\left(j\left[\arg\left(h_{d,s}^*\right) - \arg\left(h_{r,n}^* g_{ns}\right)\right]\right), \ \forall n, \quad (13)$$

and $w_s = \sqrt{P_{\max}}$.

Number of Pilot Transmissions Required and Computational Complexity: Similar to optimal rule LAS rule needs N_t pilots to obtain the direct link CSI. However, for the reflected link CSI, for each antenna k, BS only needs to know $\sum_{n=1}^{N} h_{r,n}^* g_{nk}$ to compute the selection metric. This can be estimated by configuring $x_n = 1, \ \forall n$, and sending one pilot from the user. Thus, the user only needs to send N_t pilots to obtain $\sum_{n=1}^{N} h_{r,n}^* g_{nk}$, for $k \in \{1, 2, \ldots, N_t\}$. Furthermore, to compute x_n, we need N reflected link channel gains $h_{r,1}^* g_{1s}, \ldots, h_{r,N}^* g_{Ns}$ corresponding to the antenna selected, which needs N pilots. Thus, we only need $2N_t + N$ pilots instead of $N_t + N_t N$ pilots required by the optimal AS rule. Furthermore, we need $\mathcal{O}(N_t + N)$ computations to implement LAS rule. Thus, LAS rule helps in practical implementation.

B. Subset Antenna Selection ($N_{RF} > 1$)

Here, we first present a manifold optimization based subset selection (MOBSS) algorithm to solve $\mathcal{P}$. Then, we propose a simpler alternating optimization based subset selection (AOBSS) algorithm.

For a given subset S and passive beamforming vector $\mathbf{x}$, optimal transmit beamforming vector $\mathbf{w}_{\text{opt}}$ at the BS is given by maximal ratio transmission (MRT). It is given by

$$\mathbf{w}_{\text{opt}} = \sqrt{P_{\max}} \frac{\mathbf{h}_{d,S} + \mathbf{G}_S^{\dagger}\mathbf{H}_r^{\dagger}\mathbf{x}}{\left\|\mathbf{h}_{d,S} + \mathbf{G}_S^{\dagger}\mathbf{H}_r^{\dagger}\mathbf{x}\right\|}. \quad (14)$$

Substituting this in (5), yields $P_{\max}\left\|\mathbf{x}^{\dagger}\mathbf{H}_r\mathbf{G}_S + \mathbf{h}_{d,S}^{\dagger}\right\|^2$. Thus, for a given subset S, we can find $\mathbf{x}$ that maximizes the signal power by solving the following optimization problem:

$$\mathcal{P}_S: \quad \max_{\mathbf{x}} \quad \left\|\mathbf{x}^{\dagger}\mathbf{H}_r\mathbf{G}_S + \mathbf{h}_{d,S}^{\dagger}\right\|^2, \quad (15)$$

$$\text{s.t.} \quad |x_n| = 1, \ \forall \ n = 1, \ldots, N. \quad (16)$$

The objective function in $\mathcal{P}_S$ is quadratic in $\mathbf{x}$ and is convex. However, the unit modulus constraint is non-convex. Hence, standard convex optimization techniques cannot be employed. However, the unit modulus constraint above defines a Riemannian manifold. The optimization over a manifold is locally analogous to the optimization in Euclidean space [4]. Hence, $\mathcal{P}_S$ can be solved by employing manifold optimization techniques [13]. These techniques converge to a local optimal solution by exploiting the geometry of Riemannian manifolds.

1) MOBSS Algorithm: Here, for each $S \in \mathcal{S}$, we solve $\mathcal{P}_S$ using a conjugate gradient based manifold optimization technique [4]. We obtain passive beamforming vector $\mathbf{z}_S$ and corresponding signal power. This is repeated for all possible subsets of $\mathcal{S}$. Then, the optimal subset S_{opt} is the one that yields maximum signal power and the optimal passive beamforming vector $\mathbf{x}_{\text{opt}} = \mathbf{z}_{S_{\text{opt}}}$. We then compute optimal transmit beamforming vector $\mathbf{w}_{\text{opt}}$ by substituting S_{opt} and $\mathbf{x}_{\text{opt}}$ in (14). These steps are illustrated in Algorithm 1.

Algorithm 1 Manifold Optimization Based Algorithm

1: BS estimates N_t direct link and $N_t N$ reflected link channel gains.
2: **for all** $S \in \mathcal{S}$ **do**
3: Obtain $\mathbf{z}_S$ that solves $\mathcal{P}_S$ using a conjugate gradient based manifold optimization technique.
4: **end for**
5: $S_{\text{opt}} = \arg\max_{S \in \mathcal{S}} \left\{ \left\|\mathbf{z}_S^{\dagger}\mathbf{H}_r\mathbf{G}_S + \mathbf{h}_{d,S}^{\dagger}\right\|^2 \right\}$.
6: $\mathbf{x}_{\text{opt}} = \mathbf{z}_{S_{\text{opt}}}$.
7: Compute $\mathbf{w}_{\text{opt}}$ by substituting S_{opt} and $\mathbf{x}_{\text{opt}}$ in (14).
8: **return** $S_{\text{opt}}, \mathbf{w}_{\text{opt}}, \mathbf{x}_{\text{opt}}$.

Number of Pilot Transmissions Required and Computational Complexity: Here, BS has N_{RF} RF chains. Thus, we need $\lceil N_t/N_{RF} \rceil$ number of pilots to estimate N_t direct link channel gains and $\lceil N_t/N_{RF} \rceil N$ pilots for the reflected link channel gains. MOBSS solves $\mathcal{P}_S$ for each $S \in \mathcal{S}$, which contains $\mathcal{O}({N_t}^{N_{RF}})$ elements. For each $S \in \mathcal{S}$, it solves $\mathcal{P}_S$ using the conjugate gradient based manifold optimization technique, whose worst-case complexity is $\mathcal{O}(N^{1.5})$ [4]. Hence, the computational complexity of MOBSS is $\mathcal{O}({N_t}^{N_{RF}} N^{1.5})$.

2) AOBSS Algorithm: For each antenna k, this algorithm first computes the selection metric of LAS rule in (12), i.e., $|h_{d,k}| + \left|\sum_{n=1}^{N} h_{r,n}^* g_{nk}\right|$. It then sorts them in the descending order and selects the first N_{RF} antennas from the sorted list as the subset S to transmit. Then, for the selected subset S it solves for the transmit beamforming vector $\mathbf{w}_S$ and passive beamforming vector $\mathbf{x}_S$ iteratively using the alternating optimization technique. $\mathbf{w}$ is initialized with MRT beamforming vector in the direction of the direct link, i.e., $\sqrt{P_{\max}}\mathbf{h}_{d,S}/\left\|\mathbf{h}_{d,S}\right\|$.

For a given subset S and transmit beamforming vector $\mathbf{w}$, the effective direct link channel gain from the BS to the user is given by $\mathbf{h}_{d,S}^{\dagger}\mathbf{w}$. Similarly, $[\mathbf{G}_S\mathbf{w}]_n$ is the effective channel gain from the BS to the n^{th} IRS element. For these effective channel gains, from (10), the optimal passive beamforming reflection coefficient is given by

$$x_n = \exp\left(j \arg\left(\mathbf{h}_{d,S}^{\dagger}\mathbf{w}\right) - j \arg\left(h_{r,n}^* \left[\mathbf{G}_S\mathbf{w}\right]_n\right)\right), \ \forall n. \quad (17)$$

For these reflection coefficients, the optimal $\mathbf{w}$ can be obtained by substituting them in (14). We then update $\mathbf{x}$ by substituting this $\mathbf{w}$ in (17). This iterative process is continued till the SNR improvement is less than ϵ or the maximum number of iterations M is reached. Here, in each iteration, we optimize $\mathbf{x}$ for a given $\mathbf{w}$ and then optimize $\mathbf{w}$ for the updated $\mathbf{x}$. These steps are illustrated in Algorithm 2.

Number of Pilot Transmissions Required and Computational Complexity: Similar to MOBSS algorithm, we need $\lceil N_t/N_{RF} \rceil$ pilots for the direct link CSI. However, we only need $\lceil N_t/N_{RF} \rceil$ pilots to obtain the reflected link CSI required to compute the selection metrics of the antennas and N pilots to compute the IRS reflection coefficients. This is because AOBSS algorithm uses the selection metric of LAS rule. In total, we need $2\lceil N_t/N_{RF} \rceil + N$ pilots. The computational complexity is $\mathcal{O}(N_t \log(N_t))$ to select the subset and $\mathcal{O}(N)$ per iteration to compute $\mathbf{w}$ and $\mathbf{x}$. Thus, AOBSS algorithm reduces computational complexity and the number of pilot transmissions required significantly compared to the MOBSS algorithm. Table I compares the computational complexity and number of pilot transmissions required for the proposed algorithms and SDR algorithm in [6].

IV. Numerical Results

We will now study the performance of the proposed AS rules as a function of different system parameters. We consider a uniform linear array with half-wavelength antenna spacing at the BS and a uniform planar array at the IRS. The BS and IRS are placed such that there is a dominant line-of-sight (LOS)

Algorithm 2 Alternating Optimization Based Algorithm

1: Estimate CSI required to compute the selection metrics.
2: Sort the selection metrics $|h_{d,k}| + \left|\sum_{n=1}^{N} h_{r,n}^* g_{nk}\right|$, for $k \in \{1, 2, \ldots, N_t\}$ in the descending order.
3: Assign indices of the first N_{RF} antennas in the sorted list to subset S.
4: Estimate the reflected link CSI corresponding to the subset S.
5: **Initialize** $m = 0$, $\mathbf{w}^1 = \sqrt{P_{\max}} \mathbf{h}_{d,S} / \|\mathbf{h}_{d,S}\|$.
6: **while** (SNR improvement $> \epsilon$) and ($m \leq$ M) **do**
7: Update $m = m + 1$.
8: $x_n = \exp\left(j \arg\left(\mathbf{h}_{d,S}^{\dagger} \mathbf{w}^m\right) - j \arg\left(h_{r,n}^* \left[\mathbf{G}_S \mathbf{w}^m\right]_n\right)\right)$.
9: $\mathbf{x}^m = [x_1, x_2, \ldots, x_N]$.
10: $\mathbf{w}^{m+1} = \sqrt{P_{\max}} \frac{\mathbf{h}_{d,S} + \mathbf{G}_S^{\dagger} \mathbf{H}_r^{\dagger} \mathbf{x}^m}{\left\|\mathbf{h}_{d,S} + \mathbf{G}_S^{\dagger} \mathbf{H}_r^{\dagger} \mathbf{x}^m\right\|}$.
11: **end while**
12: $\mathbf{w}_S = \mathbf{w}^{m+1}$.
13: $x_n = \exp\left(j \arg\left(\mathbf{h}_{d,S}^{\dagger} \mathbf{w}_S\right) - j \arg\left(h_{r,n}^* \left[\mathbf{G}_S \mathbf{w}_S\right]_n\right)\right)$.
14: $\mathbf{x}_S = [x_1, x_2, \ldots, x_N]$.
15: **return** S, $\mathbf{w}_S$, and $\mathbf{x}_S$.

TABLE I
COMPLEXITY COMPARISON

	Computational complexity	Pilot transmissions
Optimal AS	$\mathcal{O}(N_t N)$	$N_t + N_t N$
LAS	$\mathcal{O}(N_t + N)$	$2N_t + N$
MOBSS	$\mathcal{O}((N_t)^{N_{RF}} N^{1.5})$	$\left\lceil \frac{N_t}{N_{RF}} \right\rceil + \left\lceil \frac{N_t}{N_{RF}} \right\rceil N$
AOBSS	$\mathcal{O}(N_t \log(N_t) + N)$	$2 \left\lceil \frac{N_t}{N_{RF}} \right\rceil + N$
SDR [6]	$\mathcal{O}((N+1)^6)$	$N + 1$

component between them. Thus, the channel gain matrix from the BS to the IRS is given by

$$\mathbf{G} = \sqrt{\frac{K}{K+1}} \mathbf{G}_{\text{LOS}} + \sqrt{\frac{1}{K+1}} \mathbf{G}_{\text{NLOS}}, \tag{18}$$

where $\mathbf{G}_{\text{LOS}}$ and $\mathbf{G}_{\text{NLOS}}$ denote the LOS and Non-LOS components and K denote the Rician factor, which we set to 10. We consider independent Rayleigh fading for the direct link from the BS to the user, for the link from the IRS to the user, and $\mathbf{G}_{\text{NLOS}}$, Let d_{bi}, d_{bu}, and d_{iu} denote the distances from the BS to IRS, BS to user, and IRS to user, respectively. Corresponding path-losses are taken to be $16.6 + 22\log_{10}(d_{bi})$, $35 + 30\log_{10}(d_{bu})$, and $20 + 30\log_{10}(d_{iu})$, respectively.[1] We set $\sigma^2 = -80$ dBm, $\epsilon = 10^{-4}$, and $M = 5$.

Single Antenna Selection: Figure 2 plots the average SEP as a function of the peak transmit power $P_{\max}$ for different values of N. It compares the performance of the optimal AS rule in Result 1 with LAS rule. The average SEP decreases as $P_{\max}$ increases as the BS is allowed to transmit with a higher power. We see that LAS rule, which requires fewer pilots, is near-optimal. Also, shown is the average SEP of a single antenna system, which is significantly higher than the system with multiple antennas and one RF chain. For example, at $P_{\max} = 6$ dB, it is higher by a factor of 8.1 and 23.8, for $N = 25$ and $N = 75$, respectively. Furthermore, we see a significant reduction in the average SEP as N increases. At $P_{\max} = 6$ dB, the average SEP of the optimal AS rule for $N = 75$ is 58× lower than for $N = 25$.

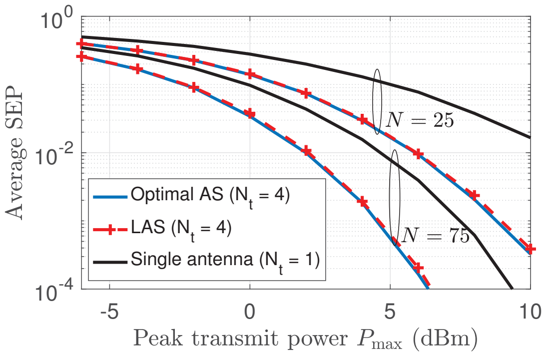

Fig. 2. Single antenna selection: Average SEP as a function of $P_{\max}$ for different number of IRS elements ($N_{RF} = 1$, $d_{bi} = 40$ m, $d_{bu} = 35$ m, $d_{iu} = 5.4$ m, and QPSK).

Subset Antenna Selection: Figure 3 plots the average receive SNR as a function of the distance between the BS and user d_{bu}. Here, the distance between the BS and IRS d_{bi} is fixed to 40 m and the user moves parallel to the line joining the BS and IRS. We compare the SNR performance of the proposed subset selection algorithms ($N_t = 4$ and $N_{RF} = 2$) with the SDR based beamforming technique that require four RF chains [6]. Also, shown is the receive SNR when there is no IRS, which decreases as d_{bu} increases. With IRS, we see that the SNR initially decreases as d_{bu} increases and then increases till $d_{bu} = 40$ m. This happens because the user moves closer to the IRS though it moves away from the BS, which makes the reflected link stronger. Furthermore, the SNR decreases for $d_{bu} > 40$ m as the user moves away from both the BS and IRS. We see that the SNR is maximum when the user is close to the IRS. We also see that the maximum benefit of increasing N occurs at $d_{bu} = 40$ m, where SNR increases by 6 dB by doubling the number of IRS elements. Thus, the placement of the IRS plays a key role in the SNR performance. Note that the simpler AOBSS algorithm performs very close to the MOBSS algorithm. We also see that the proposed subset selection algorithms can achieve SNR close to the SDR based beamforming technique with less hardware. The SNR improvement with two additional RF chains is only 1.72 dB and 2.2 dB, for $N = 50$ and $N = 100$, respectively, at $d_{bu} = 40$ m.

Figure 4 plots the average SEP as a function of $P_{\max}$ for $N_{RF} = 2$ and different values of N_t. It compares the performance of MOBSS and AOBSS algorithms. We see that the simpler AOBSS performs very close to MOBSS. Also, shown is the performance of the SDR based beamforming

[1] These are obtained for the simplified path-loss model with a signal attenuation of 30 dB at 1 m reference distance, a carrier frequency of 2.4 GHz, path-loss exponent of 2.2 for the link from the BS to IRS, and a path-loss exponent of 3 for the remaining two links. Furthermore, antenna gains at the BS, user, and IRS are taken to be 0 dB, 5 dB, 15 dB, respectively.

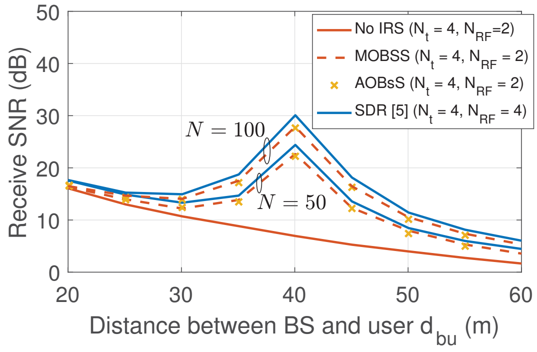

Fig. 3. The receive SNR as a function of distance from the BS to the user for different values of N ($N_t = 4$, $P_{\max} = 5$ dBm, and $d_{bi} = 40$ m).

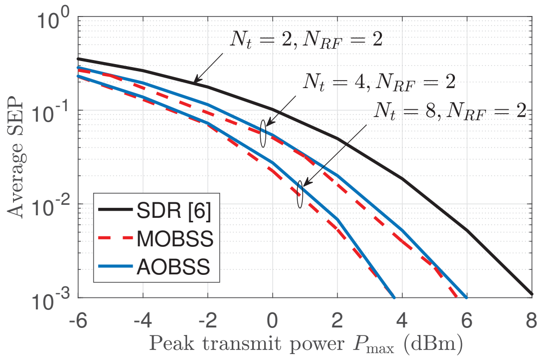

Fig. 4. Average SEP as a function of $P_{\max}$ for different number of antenna elements ($d_{bi} = 40$ m, $d_{bu} = 35$ m, $d_{iu} = 5.4$ m, $N_{RF} = 2$, $N = 25$, and QPSK).

technique [6]. With the same number of RF chains, we see that the antenna subset selection performs significantly better than the transmit beamforming. For example, at $P_{\max} = 4$ dBm, the average SEP reduces by a factor of 3.6 and 24.4 when $N_t = 4$ and $N_t = 8$, respectively, compared to $N_t = 2$.

V. Conclusions

We considered AS for an IRS assisted communication system. For it, we proposed algorithms that did joint AS, transmit beamforming at the BS, and passive beamforming at the IRS. For single AS, we showed that the optimal antenna depended only on the absolute values of the channel gains and the optimal reflection coefficient depended only on their phases. Furthermore, they were decoupled. We also proposed a simpler yet near-optimal LAS rule. For subset AS, we proposed a manifold optimization based algorithm that converges to a local optimal solution. We also proposed a simpler subset selection algorithm based on alternating optimization. It reduced the subset search complexity and the number of pilot transmissions significantly. We saw that the SNR improvement of the subset selection system is close to the system with a complete set of RF chains. We also showed that for a fixed number of RF chains the performance improves significantly as the number of reflective elements at the IRS or the antennas at the BS increase.

Appendix

A. Proof of Result 1

Here, to maximize the receive signal power at the user, the BS transmits with power $P_{\max}$ from the single antenna selected. Hence, from (8), the signal power is equal to $P_{\max}\left|h_{d,s}^* + \sum_{n=1}^{N} h_{r,n}^* g_{ns} x_n\right|^2$. Using triangle inequality, we know that

$$\left|h_{d,s}^* + \sum_{n=1}^{N} h_{r,n}^* g_{ns} x_n\right| \leq \left|h_{d,s}^*\right| + \sum_{n=1}^{N}\left|h_{r,n}^* g_{ns} x_n\right|, \quad (19)$$

where equality is achieved when $h_{r,n}^* g_{ns} x_n, \forall n$ are phase aligned with the direct link channel $h_{d,s}^*$, i.e., $\arg\left(h_{r,n}^* g_{ns} x_n\right) = \arg\left(h_{d,s}^*\right)$. Thus, for antenna s, the maximum signal power is achieved when

$$\arg\left(x_n\right) = \arg\left(h_{d,s}^*\right) - \arg\left(h_{r,n}^* g_{ns}\right), \ \forall n, \quad (20)$$

which is equal to $P_{\max}\left(\left|h_{d,s}^*\right| + \sum_{n=1}^{N}\left|h_{r,n}^* g_{ns}\right|\right)^2$. Therefore, the optimal antenna is the one that maximizes $\left|h_{d,s}^*\right| + \sum_{n=1}^{N}\left|h_{r,n}^* g_{ns}\right|$ as shown in (9). From (20), the corresponding optimal reflection coefficient is given by (10).

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