Low Complexity Passive Beamforming Algorithms for Intelligent Reflecting Surfaces with Discrete Phase-Shifts over OFDM Systems

Adit Jain, Rahul D., Salil Kashyap, *Member, IEEE*, Rimalapudi Sarvendranath, *Member, IEEE*

***Abstract*—In recent literature, intelligent reflecting surfaces (IRS) based wireless system design has been a significant point of excitement among the wireless community. We consider the problem of configuring the IRS elements efficiently and effectively for a practical IRS with discrete phase shifts and coupled elements deployed in an orthogonal frequency-division multiplexing (OFDM) based environment. We propose two near-optimal low complexity extremely scalable heuristic algorithms to design phase shifts at IRS in an OFDM system when the number of bits used to configure the IRS is limited, and the reflected channels via the IRS are spatially correlated. We benchmark the sum data rate performance of our algorithms against the theoretical upper bound and the time performance against the existing successive convex approximation. Results indicate that 4 bits are sufficient to obtain theoretically optimum sum data rates and that our proposed algorithms obtain a good trade-off between complexity and performance.**

***Index Terms*—Algorithm Design, Intelligent Reflecting Surface, Low Complexity, OFDM, Passive Beamforming**

I. Introduction

Intelligent reflecting surfaces (IRS) based wireless system design is part of active research in the domain of beyond 5G wireless communication. IRS is a software-controlled metasurface consisting of an array of passive reflection elements used to control the phase and magnitude of the incident electromagnetic signal. The setup is energy-efficient, scalable, and cost-effective and thus, can be used for information transfer in the absence of line of sight (LoS), secure communication, better wireless energy transfer, and device-to-device communication [1], [2]. They also have the benefit of decreasing the power consumption in comparison to using dedicated radio frequency (RF) chains in multiple input multiple output (MIMO) systems and improving energy efficiency [3].

Related Literature: There has been much research on channel estimation, passive beamforming, and placement of IRS in a network in recent years, as is summarized in [1]. This current work is concerned with low complexity passive beamforming design in practical IRS. Passive beamforming refers to programming the reflection coefficients of the IRS elements to obtain constructive or destructive interference at the user and meet a particular objective. The objective can be to optimize data rates, to cancel out signals for eavesdroppers or to optimally transfer energy [2], [4]. In this work, the objective is to optimize the achievable sum data rate. There has been extensive research on passive beamforming for optimizing data rates over frequency-flat channels, on both continuous and discrete phase shifts of IRS elements [5]. Different methods have been suggested for passive beamforming in order to achieve optimal data rates. Research also deals with jointly designing active and passive beamforming using a semidefinite relaxation (SDR) based approach [6]. There has also been research on jointly optimizing the IRS coefficients, and the power allocation to different users [7]. More recently, gradient projection and cross entropy-based algorithms have been suggested as more efficient solutions than directly using conventional optimization methods [8]–[10]. What has been a relatively nascent research area is optimizing phase shifts in an OFDM-based setup where the channel undergoes frequency-selective fading. While some techniques have been looked at in a continuous phase shift based IRS [6] [11], very few works have looked at discrete phase shifts [12], [13]. However, these techniques suffer on two counts, the continuous phase shifts do not capture the practicalities of deployment, and for methods on discrete phase shifts, the methods are either based on only binary reflection elements or are computationally complex for a huge number of IRS elements which are expected in practical settings.

State of the Art: Practical models involving IRS aided MIMO-OFDM based communication have been explored recently, which goes on to show that solving the optimization problem of maximizing the data rates has non-convex constraints, which makes it tough to solve [13]. Some techniques like successive convex approximation (SCA), block coordinate descent have been used to find a sub-optimal solution, however, such approaches have non-linear time complexity [1], [13]. A low complexity (log-linear) approach has been suggested but this has not been studied under IRS with discrete phase shifts to evaluate how having discrete phase shifts at the IRS affects performance [11].

While these methods help achieve the best possible data rates, they have a high computational penalty, which is not desirable since IRS would be used at the edge of the network where computing power might not be abundant. It is also desirable that the time taken to configure the phase shifts at IRS be as low as possible since the total time for training and beamforming might be limited. Having low time complexity is also desirable for situations where the user is mobile. Certain other situations and applications with strict delay constraints

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may also arise where a linear time complexity would be more desirable than attaining the best possible data rate.

Novel Contributions: In this paper, we propose a heuristic algorithm to obtain the optimal IRS phase-shifts in an OFDM system to maximize the sum data rate under discrete phase-shift constraints and spatial correlation at the IRS. For a wireless system with N IRS elements, K subcarriers and L taps, we show that the proposed algorithm gives near-optimal data rates at lower computational complexity of $\mathrm{O}(NK\log K + L + N)$ instead of $\mathrm{O}(N^{3.5}K^{4.5})$ with SDR initialized SCA (SDR-SCA) algorithm [2]. We also propose a simple vector heuristic algorithm to design phases at the IRS in linear time with a trade-off in sum data rate performance. Finally, we show that a 4-bit programmable IRS gives near-identical performance as the continuous phase-shift IRS with the proposed low complexity algorithms.

This paper contributes to the existing research in the following aspects:

1) We formulate the problem of maximizing the sum data rate of an OFDM system enabled by IRS subject to discrete phase shift constraint at the IRS over spatially correlated channels given equal power distribution across subcarriers.
2) We propose a linear time vector heuristic (VH) algorithm to obtain passive beamforming[1] coefficients for the case when the time required to beamform is constrained and the number of IRS elements is of a higher order.
3) We propose strongest tap selection (STS) based heuristic algorithm to obtain the discrete phase shifts i.e., the passive beamforming coefficients in log-linear time complexity. Note that this is a variant of the strongest tap maximization (STM) algorithm discussed in [6], [11], [14].[2]
4) We benchmark these results against the existing state of the art algorithms, namely the SDR-SCA in terms of time performance and against the theoretical upper bound in terms of sum data rate. We also analyze the impact of the number of bits used to program the IRS on the sum data rate.

Notations: In this paper, vectors are denoted by bold small letters and matrices are given by bold capital letters. $\mathbb{C}^{M\times N}$ denotes space of $M \times N$ complex valued matrices. For complex vectors $\boldsymbol{v}$, $||\boldsymbol{v}||$, $\boldsymbol{v}^T$ and $\boldsymbol{v}^H$ each denote the l_2-norm, transpose and conjugate transpose respectively. v_j denotes the j^{th} component of vector $\boldsymbol{v}$. Scalars s are denoted by normal italic letters and s^* denotes conjugate. $\odot$ denotes element wise multiplication. $\mathrm{Angle}(a)$ is the argument of the complex number a and ranges from $(-\pi, \pi]$. $\mathrm{RoundOff}(a, \mathbb{A})$ is a function which approximates a complex number a to the value b_j from discrete set of complex numbers $\mathbb{A}$ such that the angle between a and b is minimum.

$$\mathrm{RoundOff}(a, \mathbb{A}) = b_j \text{ s.t. } j = \arg\min_{b_i \in \mathbb{A}} \left| \pi + \mathrm{Angle}\left(\frac{a}{b_i}\right) \right| \tag{1}$$

$max(\boldsymbol{a})$ is a function which returns the maximum of the vector a. In case entries of $\boldsymbol{a}$ are complex, it returns the entry with the highest absolute value.

Organization of paper: The paper is organized as follows, section II presents the system model. Then, section III presents the optimization problem of maximizing the sum data rates over the subcarriers given practical IRS elements. Section IV puts forward the algorithms we suggest for passive beamforming. Section V discusses our simulation results and the quantitative computing gains in using our algorithm. Finally, section VI concludes the research undertaken and suggests further steps.

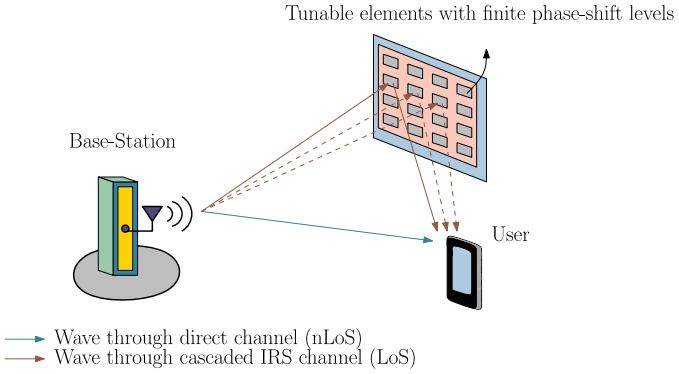


Figure 1. This is a representation of the system model considered in this paper.

II. System Model

As shown in figure 1, a single antenna base station serves a single antenna user assisted with an IRS. There are N reflection elements in the IRS setup which take indices from the set $\mathcal{N} = \{1, 2, \ldots, N\}$. The IRS elements have unit reflection gain and can provide phase shift to the incoming signal. There are M discrete phase shifts or $W = \lceil \log_2 M \rceil$ bits available to configure each IRS element. That is, the j^{th} element can take a phase shift θ_j from the set $\mathbb{A} = \{0, 2\pi\frac{1}{M}, 2\pi\frac{2}{M}, \ldots, 2\pi\frac{M-1}{M}\}$. While there is a non line-of-sight (nLoS) channel between BS and user, each of the links from BS to IRS and IRS to user is Rician faded and has a LoS component as well. The signal transmitted by the BS on the k^{th} subcarrier is given by $x_k = \sqrt{p_k}s_k$, where p_k is the power allocated to k^{th} subcarrier and the information symbols s_k is chosen such that $\mathbf{E}\left[|s_k|^2\right] = 1$. Note that $\sum_{k=1}^{K} p_k = P$ is the total power available at the BS and K is the total number of subcarriers. The received symbol y_k at the user on the k^{th} subcarrier can be written as [1]

$$y_k = (h_k^{su*} + (\boldsymbol{h_k^{si}}^H \odot \boldsymbol{h_k^{iu}})\ \boldsymbol{\Theta})\ x_k + z_k \tag{2}$$

[1]Passive Beamforming here refers to configuring the phases of the IRS to coherently add up the reflected signals in phase.

[2]The STM involves iterating over all taps and calculating the optimal IRS configuration to maximize the modulus of the effective channel gain through that tap. Then, choosing the tap and IRS configuration with the maximum modulus of the channel gain it obtains the passive beamforming coefficients. Our algorithm is different as discussed in detail in section IV.

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where $\boldsymbol{h_k^{si}}^H \in \mathbb{C}^{1\times N}$, $\boldsymbol{h_k^{iu}} \in \mathbb{C}^{1\times N}$ and $h_k^{su} \in \mathbb{C}$ denote the channel coefficients for the BS-IRS channel, the IRS-User channel and the direct channel between BS-User for the k^{th} subcarrier.[3] Furthermore, z_k is complex additive white Gaussian noise with power spectral density N_0. $\boldsymbol{\Theta} = [e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N}]^T \in \mathbb{C}^{N\times 1}$ denotes the IRS reflection coefficients where $\theta_j \in \mathbb{A}$ is the phase shift induced by the j^{th} IRS element. We introduce

$$\bar{h}_\Theta[k] = h_k^{su*} + (\boldsymbol{h_k^{si}}^H \odot \boldsymbol{h_k^{iu}})\ \boldsymbol{\Theta} \tag{3}$$

$$\bar{h}_\Theta[k] = h_k^{su*} + (\boldsymbol{h_k^{siu}})\ \boldsymbol{\Theta} \tag{4}$$

for the remaining paper as the channel coefficient for the cascaded plus the direct channel, where $\boldsymbol{h_k^{siu}} \in \mathbb{C}^{1\times N}$ is a compact representation of the cascaded channel response over the k^{th} subcarrier. The term *link* is used to describe the direct channel or a channel through a single IRS element.

III. Problem Description

The sum data rate over all subcarriers for a particular IRS configuration in our setup is expressed as

$$R = \frac{B}{K+L-1}\sum_{k=0}^{K-1}\log_2\left(1+\frac{p_k|\bar{h}_\Theta[k]|^2}{BN_o}\right) \tag{5}$$

where $\bar{h}_\theta[k] \in \mathbb{C}$ denotes the effective channel (direct link and the N links through IRS) for k^{th} subcarrier, B is bandwidth and L is the number of taps.

We study the problem of maximizing the sum data rate of an OFDM system enabled by IRS subject to the constraint that phases at the IRS are allowed to take values from a finite set of discrete phase shifts over spatially correlated channels. Considering equal power allocation across all subcarriers [4] the data rate expression in (5) gets modified to:

$$R = \frac{B}{K+L-1}\sum_{k=0}^{K-1}\log_2\left(1+\frac{P|\bar{h}_\Theta[k]|^2}{KBN_o}\right) \tag{6}$$

Mathematically, the optimization problem can be stated as

$$\begin{aligned}\mathcal{P}_1 : &\max_{\Theta}\ R\\ &\text{s.t. } \theta_j \in \mathbb{A}\ \forall j\ \in\ \mathcal{N}\end{aligned} \tag{7}$$

Here, the set $\mathbb{A} = \{0, 2\pi\frac{1}{M}, 2\pi\frac{2}{M}, \ldots, 2\pi\frac{M-1}{M}\}$ denotes the discrete phase shifts that an IRS element can potentially induce. To make the problem more tractable, instead of (7) we can consider to optimize the sum of the absolute square of the effective channel gain over different subcarriers as has been done in [2], [12], with the modified objective function as

$$R_{\text{aprx}} = \sum_{k=0}^{K-1}|\bar{h}_\Theta[k]|^2 \tag{8}$$

$$R_{\text{aprx}} = \sum_{k=0}^{K-1}|h_k^{su*} + \boldsymbol{h_k^{siu}}\Theta|^2 \tag{9}$$

$$\tag{10}$$

And the optimization problem $\mathcal{P}_1$ gets modified as,

$$\mathcal{P}_2 : \max_{\Theta}\quad R_{\text{aprx}} \tag{11}$$

$$\text{s.t.}\quad \theta_j \in \mathbb{A}\ \forall j\ \in\ \mathcal{N} \tag{12}$$

An exhaustive search could be performed to evaluate the IRS configuration which maximizes the sum data rates but this would take $O(M^N)$ and turns out to be infeasible in practical scenarios. We propose two heuristic solutions aimed at efficiently solving this and evaluate their performance.

IV. Optimizing the Passive Beamforming

Our main objective is to maximize the sum data rate of $\mathcal{P}_1$ but we instead optimize the absolute square of the effective channel gain summed over all subcarriers in $\mathcal{P}_2$. $\mathcal{P}_1$ is difficult to solve optimally even with the continuous phase shifts of IRS reflection elements [2]. A suboptimal solution could be to solve using a single statistic created using channel coefficients of the different subcarriers. We first highlight our underlying approach by considering the channel obtained by taking the mean across all subcarriers.

We then improve this algorithm by optimizing over the strongest tap in the time domain. We benchmark both the techniques against the theoretical upper bound and discuss computational complexity with the existing algorithm of successive convex approximation.

1) Vector Heuristic (VH): One representation of the channel via the j^{th} IRS element (and the direct channel) is the average over all the frequency coefficients in that channel, i.e.,

$$h_{j\ \text{mean}}^{siu} = \frac{1}{K}\sum_{k=0}^{K-1} h_{j\ k}^{siu}\ \forall\ j\in\ \mathcal{N} \tag{13}$$

$$h_{\text{mean}}^{su} = \frac{1}{K}\sum_{k=0}^{K-1} h_k^{su} \tag{14}$$

We take these average $N+1$ channel coefficients and use them to calculate the reflection coefficients required.

Intuitively these $N+1$ complex coefficients can be considered as $N+1$ two dimensional vectors. The approximate R_{aprx} term then reduces to

$$R_{\text{aprx}} = |h_{\text{mean}}^{su\ *} + \boldsymbol{h_{\text{mean}}^{siu}}\Theta|^2 \tag{15}$$

Defining $h_{\text{ref}} = max(max(\boldsymbol{h_{\text{mean}}^{siu}}), h_{\text{mean}}^{su\ *})$. Note that h_{ref} denotes the complex coefficient with the highest absolute value among the $N+1$ average values computed over K subcarriers

[3]The k^{th} component for a link is obtained by selecting the k^{th} entry of the discrete Fourier transform (DFT) of the L tap representation

[4]The power can be distributed in the different subcarriers with different p_k as well, water-filling can be used in an alternating optimization setup with passive beamforming [15].

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for the N reflected links and the direct link. And as mentioned above, $max(\boldsymbol{a})$ is a function which returns the entry with the highest absolute value. If $|\text{Angle}(\frac{h_{\text{ref}}}{h_{\text{mean}}^{su\,*}})| > \frac{\pi}{2}$, we assign $h_{\text{ref}} = -max(max(\boldsymbol{h}_{\text{mean}}^{\boldsymbol{siu}}), h_{\text{mean}}^{su\,*})$, the reasoning being that since the direct channel can't be configured, we instead align along the reference direction, which is in the same half as the direct channel. Since even a 1 bit IRS element has a phase shift of $-\pi$ possible, this alignment is always possible and will result in a better phase shift.

Further, let $\Delta_j = \text{Angle}\left(\frac{h_{j\ \text{mean}}^{siu}}{h_{\text{ref}}}\right)$ where $h_{j\ \text{mean}}^{siu}$ is the j^{th} element of $\boldsymbol{h}_{\text{mean}}^{\boldsymbol{siu}}$. This delta is simply the angle between complex numbers $h_{j\ \text{mean}}^{siu}$ and h_{ref} and we are trying to align all elements along the h_{ref}. For all j, θ_j is set to $\theta_j = \text{RoundOff}(-\Delta_j, \mathbb{A})$. The reasoning behind this comes from the passive beamforming rule of continuous phase shifts where the IRS element exactly compensates the phase difference between direct and cascaded channel. RoundOff simply rounds off the $-\Delta_j$ to the nearest value in set $\mathbb{A}$.

Simply put, all IRS elements would be aligned along the strongest mean across all subcarrier links. The main reason why we don't simply align everything along the direct channel is that for a case with very less phase shifts possible and the direct channel being weak (which is often the case with nLOS direct channel), the direct channel might not be the best possible choice to align along.

The time complexity of this algorithm is $O(NK)$. This makes this algorithm a linear algorithm in terms of the number of IRS elements and hence extremely scalable. This algorithm can be deployed in situations where the number of IRS elements are very high and when there is a time constraint on passive beamforming or in applications that are delay-sensitive.

Algorithm 1 Low Complexity Vector Heuristic Algorithm for Passive Beamforming

Input: $h_{j\ k}^{siu}, h_k^{su}\ \forall\ j, k$
Output: Θ
  $h_{j\ \text{mean}}^{siu} \leftarrow \frac{1}{K}\sum_{k=0}^{K} h_{jk}^{siu}\ \forall i$
  $\boldsymbol{h}_{\text{mean}}^{\boldsymbol{siu}} \leftarrow [h_{1\ \text{mean}}^{siu}, h_{2\ \text{mean}}^{siu} \dots h_{N\ \text{mean}}^{siu}]$
  $h_{\text{ref}} \leftarrow max(max(\boldsymbol{h}_{\text{mean}}^{\boldsymbol{siu}}), h_{\text{mean}}^{su\,*})$
  if $\left|\text{Angle}\left(\frac{h_{\text{ref}}}{h_{\text{mean}}^{su}}\right)\right| > \frac{\pi}{2}$ **then**
    $h_{\text{ref}} \leftarrow -max(max(\boldsymbol{h}_{\text{mean}}^{\boldsymbol{siu}\,H}), h_{\text{mean}}^{su\,*})$
  end if
  $\Theta \leftarrow [0, 0 \dots 0]_{1\times N}$
  for all j in $1, 2 \dots N$ **do**
    $\Delta_j = \text{Angle}\left(\frac{h_{j\ \text{mean}}^{siu}}{h_{\text{ref}}}\right)$
    $\Theta[j] \leftarrow \text{RoundOff}(-\Delta_j, \mathbb{A})$
  end for

2) Strongest-Tap Selection (STS): To further improve performance, we could consider beamforming along the strongest tap of all the N links via the IRS elements and the direct channel. As mentioned in footnote 1, a similar kind of algorithm has been suggested using strongest taps, but on continuous phase shifts [6], [11], [14]. The motivation behind this time domain transformation is that in wireless setting, the number of taps L is usually much smaller than the number of subcarriers K. Therefore, it easier to design reflection coefficients based on the time-domain representation. To obtain the time domain representation, the inverse discrete Fourier transform (IDFT) for the K channel coefficients for the $N+1$ links is taken and trimmed to first L entries. Therefore for each link, we define:

$$\boldsymbol{g}_j^{siu} = \text{IDFT}(\boldsymbol{h}_j^{siu})[1:L]\ \forall\ j\ \in\ \mathcal{N} \tag{16}$$

$$\boldsymbol{g}^{su} = \text{IDFT}(\boldsymbol{h}^{suH})[1:L] \tag{17}$$

The strongest entry out of these L taps is considered for each of the N IRS elements and these are used and processed to optimize for the passive beamforming.

$$g_{j\ \text{strong}}^{siu} = max(\boldsymbol{g}_j^{siu})\ \forall\ j\ \in\ \mathcal{N} \tag{18}$$

$$g_{\text{strong}}^{su} = max(\boldsymbol{g}^{su}) \tag{19}$$

where $max(\boldsymbol{a})$ is a function which returns the maximum of the vector a. In case entries of $\boldsymbol{a}$ are complex, it returns the entry with the highest absolute value. We use these $N+1$ coefficients, i.e., $\boldsymbol{g}_{\text{strong}}^{siu}$ and g_{strong}^{su} to obtain the N IRS configuration in a similar fashion as Algorithm 1. We find out the reference vector by taking the maximum along all $N+1$ coefficients, $g_{\text{ref}} = max(max(\boldsymbol{g}_{\text{strong}}^{siu}), g^{su})$.

Finally, similar to our approach in VH we set the IRS reflection coefficients by considering $\Delta_j = \text{Angle}\left(\frac{g_{j\ \text{strong}}^{siu}}{g_{\text{ref}}}\right)$. For all j, θ_j is set to $\theta_j = \text{RoundOff}(-\Delta_j, \mathbb{A})$. This ensures that the IRS elements discretely align the strongest taps of all links along the reference strongest tap. Note that since we only select the strongest tap, which is a scalar for all the links, we don't have to take a discrete Fourier transform and can directly align along the strongest tap.

The time complexity of the resulting algorithm 2 comes out to be $O(NK\log K + NL)$. There are $N+1$ DFT computations of complexity $O(K\log K)$ each. Then for the direct channel and each IRS element's cascaded channel we select the strongest tap which leads to an additional $O(NL)$ complexity.

V. Numerical Results and Discussion

In this section, the algorithms proposed are benchmarked against the existing state of the art solutions both in terms of date rate maximization and computation time performance or complexity.

A. Setup

As highlighted above, this is a single antenna BS and single user setup with an IRS stationed closer to the user. The IRS is stationed at the origin (0,0,0) and the BS is at (50,50,-5), making the distance between IRS and BS to be fixed at 70.88 m. The user's x and y coordinates vary uniformly between 0 to 7 m and z coordinate varies between 0 m to -4 m. There is nLoS channel between BS and User, but there is an LoS component in the channel between BS and IRS,

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Algorithm 2 Log-Linear Strongest Tap Selection Algorithm for Passive Beamforming

Input: $\boldsymbol{h}_j^{siu}, \boldsymbol{h}^{su}\ \forall\ j$
Output: Θ
$\boldsymbol{g}_j^{siu} \leftarrow \text{IDFT}(\boldsymbol{h}_j^{siu^H})[1:L]\ \forall\ j\ \in\ \mathcal{N}$
$\boldsymbol{g}^{su} \leftarrow \text{IDFT}(\boldsymbol{h}^{su^H})[1:L]$
$g_{j\ \text{strong}}^{siu} \leftarrow \max(\boldsymbol{g}_j^{siu})\ \forall\ j\ \in\ \mathcal{N}$
$g_{\text{strong}}^{su} \leftarrow \max(\boldsymbol{g}^{su})$
$\boldsymbol{g}_{\text{strong}}^{siu} \leftarrow [g_{1\ \text{strong}}^{siu}, g_{2\ \text{strong}}^{siu} \dots g_{N\ \text{strong}}^{siu}]$
$g_{\text{ref}} \leftarrow max(max(\boldsymbol{g}_{\text{strong}}^{siu}), g^{su})$
$\Theta \leftarrow [0, 0 \dots 0]_{1\times N}$
for all j in $1, 2 \dots N$ **do**
$\Delta_j = \text{Angle}\left(\frac{g_{j\ \text{strong}}^{siu}}{g_{\text{ref}}}\right)$
$\Theta[j] \leftarrow \text{RoundOff}(-\Delta_j, \mathbb{A})$
end for
return Θ

and IRS and User, i.e., each of the cascaded links via the IRS which undergoes Rician fading. The rice factor is calculated as $K_f = 13 - 0.03d$ dB where d is the distance of the link [12]. The system model is illustrated in Figure 1. Each of the L tap channels are generated using $\frac{\mu K_f}{K_f+1} G_{\text{LoS}} + \frac{\mu R^{\frac{1}{2}}}{K_f+1} G_{\text{nLoS}}$ where G_{LoS} is the LoS link, G_{nLoS}, is the nLoS link, μ is loss factor and R is the spatial correlation matrix of the IRS. Unless stated otherwise, for our simulations, we are considering a system with number of subcarriers K as 500, number of channel taps L between different links as 20, and the number of bits used to program the IRS phase shifts vary between 1 to 4 bits (i.e. 2 to 16 available phase shifts). The total power P is taken to be 30 dBm, the bandwidth B is taken as 100 MHz and N_0 is taken as -174 dBm/Hz [13]. The path loss exponent for the direct link is taken to be 3.6 and for the cascaded channels via the IRS is taken to be 2.2 [16].

The carrier frequency is taken to be 3 GHz. The separation between two IRS elements is taken to be $\lambda/2$. Our simulations also capture spatially correlated cascaded IRS channels based on the sinc correlation model as discussed in [16]. This adds to the practicality of the results. For the purpose of illustration the simulations are run assuming perfect CSI [12].

B. Benchmarking

By configuring reflection coefficients at the IRS separately for each of the subcarriers, an upper bound on the sum data rate in (7) can be obtained. The optimal reflection coefficient for each subcarrier would then be the difference in the phases of the cascaded link and the direct link on that subcarrier. The final expression for the upper bound for the sum data rate in such a scenario can be expressed as: [14]

$$R \leqslant R_{\text{UB}} = \frac{B}{K+L-1} \sum_{k=0}^{K-1} \log_2\left(1 + \frac{p_k(\bar{h}_{\text{optim}}[k])^2}{BN_o}\right) \tag{20}$$

where $\bar{h}_{\text{optim}}[k] = |h_k^{su*}| + \sum_{j=0}^{N-1} |h_{j\ k}^{siu}|$ is the optimally tuned channel coefficient for k^{th} sub carrier. The upper bound for the discrete case will certainly be lesser than or equal to this bound since the search space is now more restricted. Since, this is the best possible sum data rate that can be obtained, we use this bound for the continuous case to benchmark our results under different conditions.

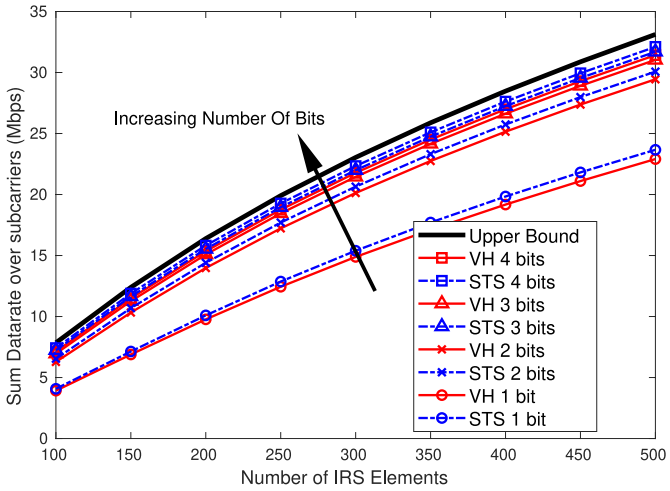


Figure 2. Benchmarking VH and STS algorithms against the upper bound and studying the effect of increasing the number of bits.

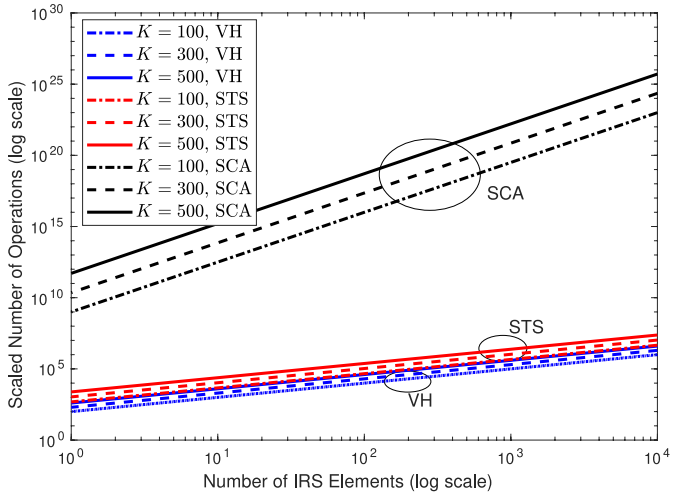


Figure 3. Theoretical worst case complexity of the STS and the VH benchmarked against SCA algorithm.

C. Results and Discussion

There are two primary results that are obtained, firstly the effect of the number of bits used to program the phase shifts at the IRS on both the proposed algorithms. Secondly, by studying the time complexity we analyse the benefits of our methods over the SCA technique and study the tradeoff for the vector based heuristic method between the time complexity and performance.

Figure 2 plots the sum data rate achieved based on the two proposed algorithms, namely, VH and STS as a function of the number of IRS elements for different number of bits

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used to configure the IRS. As a benchmark, we also plot the theoretical upper bound (based on continuous phase shifts at IRS) to assess how well the proposed algorithms perform. As expected, we observe that the sum data rate improves as we increase the number of bits, since the reflected signals align better with the direct signal as the number of levels into which we quantize the phase-shifts at IRS increases. We observe that 4 bits are sufficient with the STS technique to be close to the upper bound on the sum data rate. We also note that STS performs consistently better than VH, and STS with 3 bits of phase shifts is better than VH with 4 bits of phase shifts.

Figure 3 shows the effect of varying the number of IRS elements and subcarriers on the proportional number of operations required for the algorithm to optimize for SCA, STS and VH as derived from their theoretical worst case complexities. Looking at the corresponding time performance we see a stark improvement in the number of complex computations in comparison to the benchmark. The computational complexity of our STS algorithm increases with the order of $\mathrm{O}(NK \log K + NL)$ and VH with order of $\mathrm{O}(NK)$ with increase in IRS elements and subcarriers and SCA based algorithm increases with $\mathrm{O}(N^{3.5}K^{4.5})$ [2]. Our algorithms are certainly more scalable and deployable with both number of subcarriers and IRS elements than the SCA approach.[5] We also note that for a very large N, the VH algorithm can be applied with a tradeoff on the data rate.

VI. Conclusion and Future Work

Results show that the proposed heuristic algorithms for passive beamforming using discrete phase shifts in an OFDM based IRS aided wireless setup achieve near-optimal data rate performance even with 4 phase shift bits and have a significant advantage in terms of the time performance. This gain in time performance makes them a better choice in delay sensitive applications, in scenarios when more time for training is needed and when the user is mobile and optimal IRS phase shifts need to be determined again and again. This particular setting with discrete phase-shift programmable IRS elements of different number of bits has not been studied earlier and is of practical consequence, since during deployment IRS elements will end up having discrete phase shifts.[6] Our method is also not dependent on any initialization unlike many methods like the SCA, where number of iterations depend on the initialization strategy. Extension to a joint optimization related to power allocation across different subcarriers is straightforward as highlighted in previous research using alternating optimization with waterfilling based approach for power allocation. Although these results give an optimistic outlook, further work is needed to make this applicable, including looking into a multiuser based setup and MIMO based transmission. These additions would definitely make the optimization involved but the strongest tap and the vector heuristic technique still might prove useful in lowering the complexity of designing the phase-shifts.

[5]Although we have not benchmarked the sum data rate performance of our algorithms against SCA approach, our methods achieve near optimal performance (theoretical upper bound in (20)) with 3 to 4 phase shift bits, and the gain in time performance is order of magnitudes higher than the SCA which make our algorithms a more scalable and practical choice. Furthermore, even with SCA, the sum data rate can never exceed the theoretical upper bound in (20).

[6]The physical implementation of the IRS will involve using a single pin-diode for each bit (two-levels) of the phase shift. Since the size of these elements will be extremely small, fabricating a large number of pin-diodes may not be feasible.

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