Binary Power Control and Passive Beamforming for RIS-Assisted Spectrum Sharing Network

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***Abstract*—Underlay spectrum sharing is an effective solution to improve the spectral efficiency of wireless communications systems by allowing a secondary user (SU) to share spectrum with a primary user (PU). Meanwhile, reconfigurable intelligent surface (RIS) has been recently emerged as a promising technique to enhance energy efficiency through intelligently reconfiguring the channel environment. However, it is challenging to achieve higher quality of service for the SU under strict interference constraints to protect the PU. Therefore, adapting the transmit power of the SU and phases of RIS reflective elements while adhering to constraints on the interference caused to the PU is crucial. This adaptation is driven by the nature of interference and transmit power constraints imposed on the SU. For an RIS-assisted secondary network, we aim to minimize an average symbol error probability subject to an average interference constraint imposed by the PU. We systematically develop an optimal rule for joint binary transmit power control for the SU and passive beamforming for the RIS. We propose an algorithm to find optimal Lagrange parameter. We also propose a low-complexity coordinate descent based iterative algorithm to obtain RIS phase shifts. Simulation results demonstrate the efficacy of the optimal rule compared to several benchmarking rules.**

***Keywords*—Reconfigurable intelligent surface, spectrum sharing, binary power control, symbol error probability, average interference constraint**

I. Introduction

Spectrum sharing is a promising technology that is targeted to significantly improve the utilization of scarce wireless spectrum in the sixth-generation (6G) wireless network to support ubiquitous connectivity and diversified scenarios for satisfying the requirements of various emerging applications [1]. In an underlay mode of spectrum sharing, a secondary user (SU) can simultaneously transmit on the same frequency band as a higher priority primary user (PU) so long as the interference it causes to the primary receiver is tightly constrained [2]. However, the interference constraint results in lower reliability and limited coverage area for the SUs, which will thus limit the application potential of spectrum sharing.

Recently, reconfigurable intelligent surface (RIS) has emerged as a promising technology to achieve high energy efficiency for wireless communication systems cost-effectively [3], [4]. Specifically, RIS is a planar array consisting of a large number of passive elements, where each element is able to induce a certain phase shift, which can be programmed by a controller, to the incident electromagnetic wave. It enables passive beamforming to improve the received signal power at the SU as well as interference suppression at the PU to tackle the challenges for underlay spectrum sharing without deploying additional costly and energy-consuming communication infrastructures [5], [6].

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A. Literature: RIS-Assisted Spectrum Sharing Network (SSN)

The benefits of deploying RIS for indoor SSN is explained in [7] with an experimental setup. In [5], a joint optimization problem for transmit power control of a secondary transmitter (ST) and passive beamforming for the RIS is studied to maximize the achievable rate at a secondary receiver (SR). It considers a signal-to-interference-plus-noise ratio (SINR) constraint for a primary receiver and a peak transmit power constraint for the ST. An alternating optimization (AO)-based algorithm is proposed in it. The same optimization problem is also studied in [8] considering imperfect channel state information (CSI) and discrete RIS phase shifts.

Considering an RIS-assisted multiuser multiple-input single-output (MISO) SSN, [6] focuses on maximizing the sum data rate for the SRs by jointly optimizing the ST active beamforming and the RIS passive beamforming subject to a peak interference constraint at each primary receiver. Similar optimization problem is also studied in [9], albeit for a single SR considering both perfect and imperfect CSI. Furthermore, the work in [9] is extended for multiple RIS-assisted SSN in [10]. Instead, considering multiuser MISO SSN, [11] aims to minimize the transmit power of the ST via joint ST active beamforming and RIS passive beamforming, while meeting an SINR constraint at each SR and the peak interference constraint at each primary receiver.

B. Focus and Contributions

We focus on an RIS-assisted SSN, in which a secondary source (S) communicates with a secondary destination (D). Both S and D are equipped with a single antenna. Our objective is to develop a jointly optimal binary power control (BPC) rule at S and passive beamforming at the RIS to minimize the average symbol error probability (SEP) at D subject to an average interference constraint at the primary receiver X. Our problem formulation is novel and practical in the following aspects. Firstly, binary power control, in which S transmits with a fixed power $P_{\max}$ or with zero power, has not been

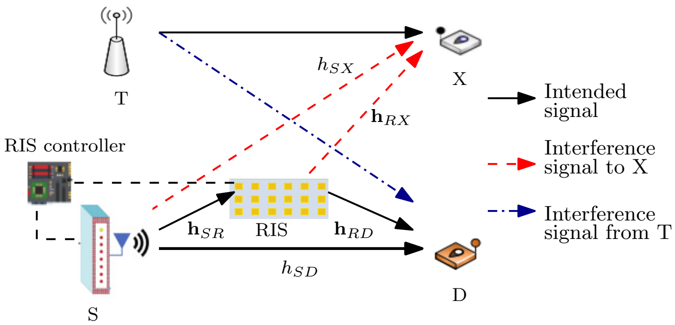


Fig. 1: An RIS-assisted underlay spectrum sharing network.

explored in the literature for RIS. BPC is practically important because it enables S to employ more energy-efficient power amplifiers and it has been studied in several wireless communication models [2], [12]. Secondly, the average interference constraint, in which the fading-averaged interference power at X due to secondary transmissions should be less than a threshold, has not been studied for RIS-assisted SSN. It is less restrictive than the conservative peak interference constraint, and, thus, enables the secondary network to perform better by changing both the RIS phase shift elements and transmit power of S depending on the channel fades. Thirdly, we focus on minimizing the average SEP unlike the achievable rate metric in [5]. SEP is an important measure of the reliability of a communication system [2], [13].

The specific contributions of this paper are as follows:

1) We systematically derive an optimal rule for joint BPC at S and passive beamforming at the RIS to minimize the average SEP at D subject to an average interference constraint at X. This involves solving a system of non-linear equations, where the number of equations is equal to the number of RIS elements. To reduce this complexity, we propose a coordinate descent algorithm (CDA) to obtain the RIS phase shifts.
2) We provide exact closed form expressions for the average interference power at X when RIS phases are adjusted to achieve (i) minimum SEP at D and (ii) minimum interference power at X. These results help characterizing the optimal rule.
3) Our simulation results demonstrate the behaviour of the optimal rule for different values of interference threshold. It shows the effect of RIS placement and reveals that the optimal rule significantly reduces the average SEP compared to other benchmarking rules.

II. System Model

As shown in Figure 1, we consider an RIS-assisted underlay spectrum sharing network, in which the secondary network consists of a secondary source S, a secondary destination D, and an RIS having M reflective elements. It coexists with a primary network that consists of a primary transmitter T and a primary receiver X. We assume that all nodes are equipped with a single antenna. The complex baseband channel gains for S-D and S-X links are denoted by h_{SD} and h_{SX}, respectively. We consider independent Rayleigh fading for these links. Therefore, $h_{SD} \sim \mathcal{CN}(0, \mu_{SD})$ and $h_{SX} \sim \mathcal{CN}(0, \mu_{SX})$, where μ_{SD} and μ_{SX} denote their mean channel power gains, respectively. Here, $h \sim \mathcal{CN}(0, \mu)$ means that h is a circularly symmetric complex Gaussian random variable (RV) with zero mean and variance μ.

In reality, the RIS is to be deployed such that there exists a line of sight (LoS) path in the RIS-reflected links. We further assume a worst case scenario in which the LoS path also exists between the RIS and X. Therefore, we model the complex baseband channel gains for S-RIS, RIS-D, and RIS-X links as Rician fading following [9], [10], [14]. These baseband channels gains for the m^{th} RIS reflective element, for $1 \leq m \leq M$, are respectively represented by

$$h_{SR}^m = \sqrt{\mu_{SR}} \left(\sqrt{\frac{K_{SR}}{1+K_{SR}}} \bar{h}_{SR}^m + \sqrt{\frac{1}{1+K_{SR}}} \tilde{h}_{SR}^m \right), \quad (1)$$

$$h_{RD}^m = \sqrt{\mu_{RD}} \left(\sqrt{\frac{K_{RD}}{1+K_{RD}}} \bar{h}_{RD}^m + \sqrt{\frac{1}{1+K_{RD}}} \tilde{h}_{RD}^m \right), (2)$$

$$h_{RX}^m = \sqrt{\mu_{RX}} \left(\sqrt{\frac{K_{RX}}{1+K_{RX}}} \bar{h}_{RX}^m + \sqrt{\frac{1}{1+K_{RX}}} \tilde{h}_{RX}^m \right), (3)$$

where K_{SR}, K_{RD}, and K_{RX} are the Rician factors and μ_{SR}, μ_{RD}, and μ_{RX} denote the mean channel power gains of the S-RIS, RIS-D, and RIS-X links, respectively. Furthermore, $\bar{h}_{SR}^m$, $\bar{h}_{RD}^m$, and $\bar{h}_{RX}^m$ represents the fixed channel related to the LoS component, while $\tilde{h}_{SR}^m$, $\tilde{h}_{RD}^m$, and $\tilde{h}_{RX}^m$ represents the non-LoS channel following independent $\mathcal{CN}(0,1)$ distribution.

Let $\theta_{h_{SX}}, \theta_{h_{SD}}, \theta_{h_{SR}^m}, \theta_{h_{RD}^m}, \theta_{h_{RX}^m}$ denote the phases of the corresponding baseband channel gains h_{SX}, h_{SD}, h_{SR}^m, h_{RD}^m, and h_{RX}^m, for $1 \leq m \leq M$, respectively. The RIS reflection coefficient can be expressed as $\Phi_m = \beta_m e^{j\theta_m}$, where $\beta_m \in [0,1]$ is the reflection loss and $\theta_m \in [0, 2\pi)$ and the phase shift of the m^{th} RIS element. To maximize the reflection power of the RIS, we fix $\beta_m = 1$, for all m, as typically assumed in [11]. Due to high path loss, the power of the signals that are reflected by the RIS two or more times is ignored [5]. Let $\boldsymbol{\Theta} \triangleq [\theta_1, \theta_2, \ldots, \theta_M]$ denote the RIS phase shift vector.

A. Data Transmission and Interference Model

Let S transmits a data symbol x with $\mathbb{E}\left[|x|^2\right] = 1$. The transmit power of S is denoted by $P_S \in \{P_{\max}, 0\}$, following a BPC rule. We assume quasi-static flat-fading channel model. Furthermore, the RIS is always assumed to be ON during data transmission from S to D. The combined received signal at D by the direct link and RIS-reflected link is then given by $y_D = \sqrt{P_S} \left(h_{SD} + \sum_{m=1}^{M} h_{RD}^m e^{j\theta_m} h_{SR}^m \right) x + n_D + n_P$, where, $n_D \sim \mathcal{CN}(0, \sigma_D^2)$ is the Gaussian noise at D and $n_P \sim \mathcal{CN}(0, \sigma_P^2)$ is the interference at D due to transmissions by T via the direct link, which is modeled as Gaussian [2], [10].[1] We further assume that RIS-reflected interference signal at D due to transmissions by T is negligible. This may happen

[1] The Gaussian interference assumption is a tractable and worst case model for the interference [2], [15]. It is valid when the primary transmitters are far away from D [9], [11]. Even with one T, this is justified when T transmits OFDM signal and provides SEP upper bound for a constant amplitude signal.

when RIS is located close to S. Thus, $n_D + n_P \sim \mathcal{CN}(0, \sigma^2)$, where $\sigma^2 \triangleq \sigma_D^2 + \sigma_P^2$. The instantaneous SINR $\gamma_S(P_S, \boldsymbol{\Theta})$ at D can now be expressed as

$$\gamma_S(P_S, \boldsymbol{\Theta}) = \frac{P_S}{\sigma^2} \left| h_{SD} + \sum_{m=1}^{M} h_{RD}^m e^{j\theta_m} h_{SR}^m \right|^2. \quad (4)$$

The interference signal i_X received at X due to transmissions by S via direct link and RIS-reflected link is given by $i_X = \sqrt{P_S}\left(h_{SX} + \sum_{m=1}^{M} h_{RX}^m e^{j\theta_m} h_{SR}^m\right) x$. Let $I(P_S, \boldsymbol{\Theta})$ denote the instantaneous interference power at X. It can be written as

$$I(P_S, \boldsymbol{\Theta}) = P_S \left| h_{SX} + \sum_{m=1}^{M} h_{RX}^m e^{j\theta_m} h_{SR}^m \right|^2. \quad (5)$$

The source is subject to an average interference constraint by ensuring that the fading-averaged interference power at X due to transmissions by S is less than or equal to a threshold I_{th}, i.e., $\mathbb{E}[I(P_S, \boldsymbol{\Theta})] \le I_{\text{th}}$. The choice of I_{th} is determined by the quality of service requirements of the primary network.

B. Problem Statement

Our objective is to minimize the average SEP of the secondary network. Let SEP(γ_S) denote the instantaneous SEP at D, which is a function of received SINR γ_S. From [13, (14)], for all modulation techniques, it can be written in the following generic form SEP$(\gamma_S) \approx c_1 \exp(-c_2\gamma_S)$, where c_1 and c_2 are modulation dependent constants. Our goal is to jointly optimize source's transmit power P_S and the RIS phase shift vector $\boldsymbol{\Theta}$ in order to minimize the average SEP of the secondary network subject to the average interference constraint. To keep the notation simple, we do not explicitly show the dependence of P_S and $\boldsymbol{\Theta}$ on the various channel fades. It can be mathematically stated as the following stochastic, constrained optimization problem $\mathcal{P}$:

$$\mathcal{P}: \min_{P_S, \boldsymbol{\Theta}} \mathbb{E}[c_1 \exp(-c_2\gamma_S(P_S, \boldsymbol{\Theta}))], \quad (6)$$

$$\text{s.t. } \mathbb{E}[I(P_S, \boldsymbol{\Theta})] \le I_{\text{th}}, \quad (7)$$

$$P_S \in \{P_{\max}, 0\}. \quad (8)$$

III. Analysis of Average Interference Power

To solve $\mathcal{P}$, we now derive closed-form expressions for the average values of the minimum interference powers at X when S transmits with power $P_{\max}$. We further derive the average interference power at X to obtain the minimum average SEP at D when no interference constraint is active.

A. Fading-averaged minimum interference power

When S transmits with power $P_S = P_{\max}$, the primary receiver will experience the minimum interference when the direct S-X link is 180° out of phase with the composite S-RIS-X link. This happens when $\theta_m = \pi + \theta_{h_{SX}} - \left(\theta_{h_{SR}^m} + \theta_{h_{RX}^m}\right)$, for $1 \le m \le M$. Under this, the cross term in (5) $\sum_{m=1}^{M} h_{RX}^m e^{j\theta_m} h_{SR}^m h_{SX}^* = -\sum_{m=1}^{M} |h_{RX}^m||h_{SR}^m||h_{SX}|$.

Lemma *1:* The fading-averaged value of the minimum interference power $I_{\text{avg}}^{\min}$ at X for $P_S = P_{\max}$ is given by

$$I_{\text{avg}}^{\min} = \mathbb{E}\left[P_{\max}\left(|h_{SX}| - \sum_{m=1}^{M} |h_{RX}^m||h_{SR}^m|\right)^2\right] = A_1 + A_2 - A_3. \quad (9)$$

where, $A_1 = P_{\max}\mu_{SX}$, $A_2 = P_{\max}M\mu_{SR}\mu_{RX} + \frac{P_{\max}M(M-1)\pi^2\mu_{SR}\mu_{RX}}{16(K_{SR}+1)(K_{RX}+1)}\left(L_{\frac{1}{2}}(-K_{SR})L_{\frac{1}{2}}(-K_{RX})\right)^2$, and $A_3 = P_{\max}M\sqrt{\frac{\pi^3\mu_{SX}\mu_{SR}\mu_{RX}}{16(K_{RX}+1)(K_{SR}+1)}}L_{1/2}(-K_{SR})L_{1/2}(-K_{RX})$, with $L_{\frac{1}{2}}(.)$ denotes the Laguerre Polynomial [16, (22.2.13)].

Proof: The proof is relegated to Appendix A. ■

B. Unconstrained average interference power ($I_{th} = \infty$)

Let I_{un} denote the average interference power at X to obtain the minimum average SEP at D when no interference constraint is active, i.e., $I_{\text{th}} = \infty$. It happens when $P_S = P_{\max}$ and $\theta_m = \theta_{h_{SD}} - \left(\theta_{h_{SR}^m} + \theta_{h_{RD}^m}\right)$, for $1 \le m \le M$.

Lemma *2:* The fading-averaged value of the interference power I_{un} at X to obtain the minimum average SEP at D, when no interference constraint is active is given by

$$I_{\text{un}} = P_{\max}\Bigg(\mu_{SX} + M\mu_{SR}\mu_{RX} + \frac{M(M-1)\pi^2\mu_{SR}\mu_{RX}}{16(K_{SR}+1)(K_{RX}+1)} \times \left[L_{1/2}(-K_{SR})L_{1/2}(-K_{RX})\zeta(K_{RX})\zeta(K_{RD})\right]^2\Bigg), \quad (10)$$

where, $\zeta(K) = \sqrt{\frac{K}{\pi}}\int_0^{\pi}\cos^2\theta e^{-K\sin^2\theta}\left(1 + \text{erf}\left(\sqrt{K}\cos\theta\right)\right)d\theta$ and erf$(.)$ denotes the error function [16, (7.1.1)].

Proof: The proof is relegated to Appendix B. ■

IV. Optimal BPC and RIS Passive Beamforming

We now systematically derive an optimal rule for joint BPC at S and passive beamforming at the RIS that solves $\mathcal{P}$ using the above Lemmas. Let $\bar{P}_S \in \{P_{\max}, 0\}$ denote the optimal transmit power of S and $\bar{\boldsymbol{\Theta}} = \left[\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_M\right]$ denote the optimal phase shift vector of the RIS reflective elements. A feasible rule is the one that satisfies the average interference constraint in (7). We want to find $\bar{P}_S$ and $\bar{\boldsymbol{\Theta}}$ that minimizes the average SEP at D while satisfying the average interference constraint at X. Let SEP$(P_S, \boldsymbol{\Theta})$ denote the instantaneous SEP for a specific choice of P_S and $\boldsymbol{\Theta}$. It is given by the term inside the expectation operator in (6). We characterize the optimal rule as follows:

- *Region I ($I_{th} \ge I_{un}$)*: The optimal values of P_S and $\boldsymbol{\Theta}$ that minimizes the average SEP at D while satisfying the average interference constraint is given by

$$\bar{P}_S = P_{\max} \text{ and } \bar{\boldsymbol{\Theta}} = \theta_{h_{SD}}\mathbf{1} - \left(\boldsymbol{\Theta}_{h_{SR}} + \boldsymbol{\Theta}_{h_{RD}}\right), \quad (11)$$

  where $\boldsymbol{\Theta}_{h_{SR}} \triangleq \left[\theta_{h_{SR}^1}, \theta_{h_{SR}^2}, \ldots, \theta_{h_{SR}^M}\right]$, $\boldsymbol{\Theta}_{h_{RD}} \triangleq \left[\theta_{h_{RD}^1}, \theta_{h_{RD}^2}, \ldots, \theta_{h_{RD}^M}\right]$, and $\mathbf{1} \triangleq [1, 1, \ldots, 1]$. We shall refer the rule in (11) as the *unconstrained rule* henceforth. The average interference caused by it is given in (10).
- *Region II ($I_{avg}^{\min} \le I_{th} < I_{un}$)*: The unconstrained rule is not a feasible rule in this region as $I_{\text{un}} > I_{\text{th}}$. Therefore,

it cannot be optimal. The following result completely characterizes the optimal rule in this region.

Result *1:* The optimal BPC and RIS beamforming rule that minimizes the average SEP under the average interference constraint in Region II is $\bar{P}_S = P_{\max}$ and

$$\bar{\mathbf{\Theta}} = \underset{\mathbf{\Theta}}{\operatorname{argmin}} \left\{ \text{SEP}\left(P_{\max}, \mathbf{\Theta}\right) + \lambda I\left(P_{\max}, \mathbf{\Theta}\right) \right\}, \quad (12)$$

where λ is the Lagrange parameter. In this region, $\lambda > 0$. The value of λ is computed such that the secondary network satisfies the average interference constraint with *equality*. Therefore, $I_{\text{avg}}(\lambda) \triangleq \mathbb{E}\left[I\left(P_{\max}, \bar{\mathbf{\Theta}}\right)\right] = I_{\text{th}}$.

Proof: The proof is relegated to Appendix C. ■

We note that $I_{\text{avg}}(0) = I_{\text{un}}$ and it can be shown that $I_{\text{avg}}(\lambda)$ is a continuous and monotonically decreasing function of λ for $\lambda > 0$. Therefore, the bisection search method can be used to find the optimal value for λ in this case. The algorithm to compute optimal value for λ and $\bar{\mathbf{\Theta}}$ is summarized in Algorithm 1.

Algorithm 1 Algorithm to obtain Optimal λ and $\bar{\mathbf{\Theta}}$

1: **Initialize** $\lambda = \lambda_l = \lambda_u = 0$
2: Until $I_{\text{avg}}(\lambda) \leq I_{\text{th}}$, find λ_u by updating $\lambda_u \leftarrow \lambda + 0.01$ and obtaining $\bar{\mathbf{\Theta}}$ using (12) for $\lambda = \lambda_u$.
3: **while** $|I_{\text{avg}}(\lambda) - I_{\text{th}}| > \epsilon$ **do**
4: Calculate $\lambda = \frac{\lambda_l + \lambda_u}{2}$. Find $\bar{\mathbf{\Theta}}$ by using (12).
5: Compute $I_{\text{avg}}(\lambda) = \mathbb{E}\left[I\left(P_{\max}, \bar{\mathbf{\Theta}}\right)\right]$.
6: If $I_{\text{avg}}(\lambda) < I_{\text{th}}$, $\lambda_u \leftarrow \lambda$; otherwise $\lambda_l \leftarrow \lambda$.
7: **end while**
8: **return** The optimal solutions for λ and $\bar{\mathbf{\Theta}}$.

- *Region III* ($I_{th} < I_{avg}^{\min}$): In this region, setting $P_S = P_{\max}$ and $\theta_m = \pi + \theta_{h_{SX}} - \left(\theta_{h_{SR}^m} + \theta_{h_{RX}^m}\right) \triangleq \theta_m^{\min}$ for all the possible channel fades violates the interference constraint. Let $g \triangleq \left(|h_{SX}| - \sum_{m=1}^{M} |h_{RX}^m||h_{SR}^m|\right)^2$ denotes the equivalent channel power gain from S to X for the minimum interference condition using $\theta_m = \theta_m^{\min}$, for $1 \leq m \leq M$. Furthermore, let $\beta > 0$ be the unique solution of the equation $P_{\max} \int_0^{\beta} g f_g(g)\, dg = I_{\text{th}}$, where $f_g(g)$ denotes the probability density function (PDF) of g. Then, the optimal power $\bar{P}_S = P_{\max}$, if $g \leq \beta$; otherwise, $\bar{P}_S = 0$. The parameter β is computed numerically. The optimal phase shift vector $\bar{\mathbf{\Theta}} = (\pi + \theta_{h_{SX}})\,\mathbf{1} - (\mathbf{\Theta}_{h_{SR}} + \mathbf{\Theta}_{h_{RX}})$. The parameter β can be derived analytically similar to techniques used in Result 1 of [2] and is not included here due to space constraints.

Obtaining $\bar{\mathbf{\Theta}}$ *from* (12)*:* By taking partial derivative of the objective function $\text{SEP}\left(P_{\max}, \mathbf{\Theta}\right) + \lambda I\left(P_{\max}, \mathbf{\Theta}\right)$ w.r.t. θ_m, and setting it to zero, we get the following equations for $1 \leq m \leq M$:

$$a_m Y_m + a_m^* Y_m^* + \sum_{\substack{l=1 \\ l \neq m}}^{M} C_{ml} Y_m Y_l^* + C_{ml}^* Y_m^* Y_l$$
$$= \frac{c_1 c_2}{\lambda \sigma^2} \exp\left(-\frac{c_2 P_{\max}}{\sigma^2}\left|h_{SD} + \sum_{m=1}^{M} h_{RD}^m Y_m h_{SR}^m\right|^2\right)$$
$$\times \left(b_m Y_m + b_m^* Y_m^* + \sum_{\substack{l=1 \\ l \neq m}}^{M} D_{ml} Y_m Y_l^* + D_{ml}^* Y_m^* Y_l\right),$$

where $a_m = j h_{SR}^m h_{RX}^m h_{SX}^*$, $b_m = j h_{SR}^m h_{RD}^m h_{SD}^*$, $C_{ml} = j h_{SR}^m h_{RX}^m h_{SR}^{l*} h_{RX}^{l*}$, $D_{ml} = j h_{SR}^m h_{RD}^m h_{SR}^{l*} h_{RD}^{l*}$, and $Y_m = e^{j\theta_m}$. Finding optimal $\bar{\mathbf{\Theta}}$ involves solving a system of non-linear equations with M unknowns and M equations. Efficient multi-step iterative methods, e.g., Newton-Raphson's method (complexity $\mathcal{O}(kM^3)$ for k iterations) and hybrid Newton-Raphson and Stochastic Gradient Descent method (complexity $\mathcal{O}(kM)$) are available to handle this problem [17].

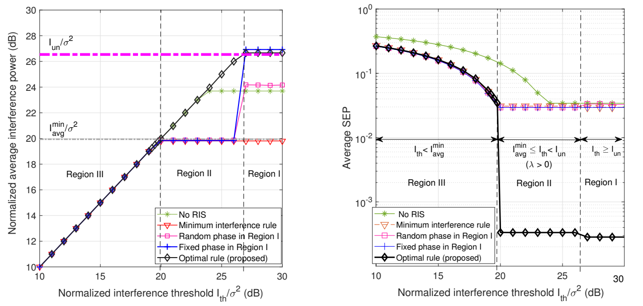
We next propose a suboptimal coordinate descent algorithm to obtain RIS phases with lower complexity. At each iteration of CDA, we first minimize the metric w.r.t. each phase shift θ_m while the other phase shifts are kept fixed. Since θ_m is the solution of the above equation, it can be easily computed using fsolve in Matlab. The computational complexity of CDA is $\mathcal{O}(M)$ per iteration as we need to solve for M phase shifts one by one.

Discussion on CSI assumptions: Channel estimation is necessary for implementing the optimal rule. Considering the passive nature of the RIS, we adopt the time-division duplex protocol to exploit channel reciprocity and reduce CSI feedback overhead. Following the channel estimation protocol in [10], S needs to know perfect knowledge about the direct S-D channel gain h_{SD} and the cascaded channel gain matrix $\mathbf{H}_{RD}\mathbf{h}_{SR}$ of S-RIS-D link, where $\mathbf{H}_{RD} = \operatorname{diag}\left(\mathbf{h}_{RD}\right)$, $\mathbf{h}_{RD} = [h_{RD}^1, \ldots, h_{RD}^M]^T$, and $\mathbf{h}_{SR} = [h_{SR}^1, \ldots, h_{SR}^M]^T$, whose details have been omitted for brevity. Similarly, the direct S-X channel gain h_{SX} and the cascaded channel gain matrix of S-RIS-X link can be estimated. The source computes $\mathbf{\Theta}$ based on this CSI and communicates to the RIS controller through a control link.

V. Numerical Results

In this section, simulation results are provided to evaluate the performance of the proposed scheme. For any two nodes A $\in \{S, \text{RIS}\}$ and B $\in \{D, \text{RIS}, X\}$, the mean channel power gain is given by $\delta\left(d_0/d_{\text{AB}}\right)^{\alpha_{\text{AB}}}$ [11]. Here, $\delta = -30$ dB is the path loss at reference point $d_0 = 1$ m, d_{AB} denotes the distance between the nodes A and B, and α_{AB} denotes the path loss exponent, which is considered as 3.5 for the direct S-D and S-X channels and 2.5 for the RIS-assisted channels. We set the Rician factors as 2, the sum of thermal noise power and interference power from T as $\sigma^2 = -100$ dBm, and $\epsilon = 10^{-4}$. Unless mentioned otherwise, we set $d_{SD} = 100$ m, $d_{SX} = 95.5$ m, $d_{SR} = 98.5$ m, $d_{RD} = 10.2$ m, and $d_{RX} = 3$ m.

Behaviour of the proposed rule and bechmarking: Fig. 2(a) and Fig. 2(b) plot the normalized average interference power at X in dB and the average SEP at D, respectively, by the optimal rule as a function of I_{th}/σ^2. Then, the proposed scheme is compared with several benchmarking methods satisfying the average interference constraint: (1) Minimum interference rule, in which we use $\theta_m = \theta_m^{\min}$ in Region I; (2) Fixed phase rule, in which all values of the RIS phase shifts are kept fixed in Region I; (3) Random phase rule, in which the phase of each reflective element is randomly and uniformly generated between 0 and 2π in Region I. For the above three rules, we set $P_S = P_{\max}$ in Region I and II. Further, $\theta_m = \theta_m^{\min}$ is used in



(a) Average interference power

(b) Average SEP

Fig. 2: Normalized average interference power at X and average SEP at D as a function of I_{th}/σ^2 by the various rules ($M = 20$, $P_{\max} = 23$ dBm and 8PSK).

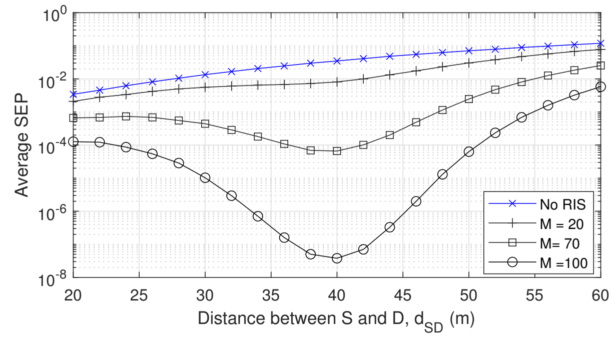


Fig. 3: Average SEP as a function of distance between S and D ($P_{\max} = 5$ dBm, $I_{\text{th}}/\sigma^2 = 20$ dB, and 8PSK).

Region II. However, in Region III, we adopt the same strategy for power control and RIS beamforming as the optimal rule. We also compare with no-RIS case, in which the optimal BPC rule given in [2] is used.

We first discuss the behaviour of the proposed scheme. (a) When $I_{\text{th}}/\sigma^2 < I_{\text{avg}}^{\min}/\sigma^2$ ($= 20$ dB): In this Region III, as I_{th}/σ^2 increases, the normalized average interference power ($= I_{\text{th}}/\sigma^2$) increases and the average SEP decreases. The optimal power control policy decides to transmit with full power $P_{\max}$ more often as I_{th}/σ^2 increases. (b) When $I_{\text{avg}}^{\min}/\sigma^2 \le I_{\text{th}}/\sigma^2 < I_{\text{un}}/\sigma^2$: In this Region II, the normalized average interference power equals I_{th}/σ^2 and continues to increase as I_{th}/σ^2 increases. The average SEP decreases abruptly from the Region III and becomes almost independent of I_{th}/σ^2. This is because in order to meet the average interference constraint with equality, the value of λ (> 0) decreases as I_{th}/σ^2 increases, and S transmits with fixed power $P_{\max}$ for all channel fades. (c) When $I_{\text{th}}/\sigma^2 \ge I_{\text{un}}/\sigma^2$ ($= 26.5$ dB): In this Region I, the unconstrained rule is the optimal. The normalized average interference power saturates at I_{un}/σ^2. The average SEP decreases slightly and becomes independent of I_{th}/σ^2 as in a conventional non-SSN. Note that no RIS scheme incurs the highest SEP. In Region III, the average SEP and interference power of all other benchmarking schemes are the same as those of the optimal rule by construction. However, for Region II and I, the SEP of the proposed scheme outperforms all the other schemes by two orders of magnitude.

Effect of RIS placement: Fig. 3 plots the average SEP of the optimal rule as a function of S-D distance d_{SD} for different values of M. The locations of S, X, and RIS are set as $[0, 0, 0]$, $[60, 0, 0]$, and $[40, 10, 0]$, respectively, and d_{SD} is varying as D moves towards S. We also plot the average SEP for no RIS case [2]. It increases as d_{SD} increases. However, using RIS with $M = 20$, we see that the average SEP initially increases as d_{SD} increases and then starts decreasing till $d_{SD} = 40$ m. This is due to the fact that although D moves away from S, it reaches closer to the RIS, which results a stronger RIS-reflected link. Furthermore, the average SEP increases with d_{SD} for $d_{SD} > 40$ m. This is because D moves away from both S and the RIS. We further see that as M increases, the average SEP decreases and the maximum benefit is observed when D is close to RIS ($d_{SD} = 40$ m). Thus, the placement of the RIS plays a crucial role.

VI. Conclusions

Considering an RIS-assisted spectrum sharing network, we developed novel SEP-optimal rule for binary transmit power control for the source and passive beamforming for the RIS under an average interference constraint. We proposed an algorithm to accurately compute optimal Lagrange parameter. We also proposed a low-complexity suboptimal CDA to obtain the RIS phase shift vector. We derived exact closed-form expressions for the average interference power when RIS phases are adjusted to obtain (i) minimum SEP at D and (ii) minimum interference power at X. We observed that the optimal rule reduced the average SEP by upto two orders of magnitude compared to suboptimal rules for joint BPC and RIS passive beamforming. We showed that as the number of RIS reflective elements increased, the average SEP decreased, and the maximum benefit was observed when D was close to RIS. Including continuous power control to better utilize the available CSI can be potential future research.

Appendix

A. Proof of Lemma 1

Using $P_S = P_{\max}$ and $\theta_m = \pi + \theta_{h_{SX}} - \left(\theta_{h_{SR}^m} + \theta_{h_{RX}^m}\right)$ in (5) and simplifying further, we get the minimum value of the instantaneous interference power as $P_{\max}\left(|h_{SX}| - \sum_{m=1}^{M} |h_{RX}^m||h_{SR}^m|\right)^2$. The minimum average interference power is given by $I_{\text{avg}}^{\min} = A_1 + A_2 - A_3$, where $A_1 = P_{\max}\mathbb{E}\left[|h_{SX}|^2\right] = P_{\max}\mu_{SX}$, $A_2 = P_{\max}\mathbb{E}\left[\left(\sum_{m=1}^{M} |h_{RX}^m|\,|h_{SR}^m|\right)^2\right]$, and $A_3 = 2P_{\max}\mathbb{E}\left[|h_{SX}|\sum_{m=1}^{M} |h_{RX}^m|\,|h_{SR}^m|\right]$.

Now, A_2 simplifies to $A_2 = P_{\max}\mathbb{E}[\sum_{m=1}^{M} |h_{RX}^m|^2\,|h_{SR}^m|^2 + \sum_{m=1}^{M}\sum_{n=1, n\neq m}^{M} |h_{RX}^m|\,|h_{SR}^m|\,|h_{RX}^n|\,|h_{SR}^n|]$. It is easy to see that $\mathbb{E}\left[\sum_{m=1}^{M} |h_{RX}^m|^2\,|h_{SR}^m|^2\right] = M\mu_{RX}\mu_{SR}$. For Rician fading, $\mathbb{E}\left[|h_{SR}|\right] = \sqrt{\frac{\pi\mu_{SR}}{4(K_{SR}+1)}}L_{1/2}\left(-K_{SR}\right)$ and $\mathbb{E}\left[|h_{RX}|\right] = \sqrt{\frac{\pi\mu_{RX}}{4(K_{RX}+1)}}L_{1/2}\left(-K_{RX}\right)$, and observing the fact that $|h_{SR}^m|$ is independent of $|h_{RX}^m|$ and $|h_{SR}^n|$, for $n \neq m$, $\mathbb{E}\left[\sum_{m=1}^{M}\sum_{n=1,n\neq m}^{M}|h_{RX}^m|\,|h_{SR}^m|\,|h_{RX}^n|\,|h_{SR}^n|\right]$ $= \frac{M(M-1)\pi^2\mu_{SR}\mu_{RX}}{16(K_{SR}+1)(K_{RX}+1)}\left(L_{1/2}(-K_{SR})\,L_{1/2}(-K_{RX})\right)^2$.

Under Rayleigh fading, we can show that $\mathbb{E}\left[|h_{SX}|\right] = \frac{\sqrt{\pi\mu_{SX}}}{2}$. Using the independent channel property, we get $A_3 = P_{\max}M\sqrt{\frac{\pi^3\mu_{SX}\mu_{SR}\mu_{RX}}{16(K_{RX}+1)(K_{SR}+1)}}L_{1/2}\left(-K_{RX}\right)L_{1/2}\left(-K_{SR}\right)$.

Combining A_1, A_2, and A_3 together yields $I_{\text{avg}}^{\min}$ in (9).

B. Proof of Lemma 2

To minimize the average SEP under no interference constraint, we set $P_S = P_{\max}$ and $\theta_m = \theta_{h_{SD}} - \left(\theta_{h_{SR}^m} + \theta_{h_{RD}^m}\right)$. Expressing $|Z|^2$ by ZZ^* in (5) and averaging over channel fades, the average interference power is given by

$$
\begin{aligned}
I_{\text{un}} = P_{\max}\mathbb{E}\Bigg[&|h_{SX}|^2 + h_{SX}^*\sum_{m=1}^{M}h_{RX}^m e^{j\theta_m}h_{SR}^m \\
&+h_{SX}\sum_{m=1}^{M}\left(h_{RX}^m\right)^* e^{-j\theta_m}\left(h_{SR}^m\right)^* + \sum_{m=1}^{M}\left|h_{RX}^m\right|^2\left|h_{SR}^m\right|^2 \\
&+\sum_{m=1}^{M}\sum_{\substack{n=1\\ n\neq m}}^{M}h_{RX}^m h_{SR}^m\left(h_{RX}^n\right)^*\left(h_{SR}^n\right)^* e^{j(\theta_m-\theta_n)}\Bigg], \\
&\triangleq T_1 + T_2 + T_3 + T_4 + T_5. \quad (13)
\end{aligned}
$$

Note that $T_1 = P_{\max}\mu_{SX}$. Since the direct S-X link is independent of the cascaded S-RIS-X link and $\mathbb{E}\left[h_{SX}\right] = 0$, we have $T_2 = 0$ and $T_3 = 0$. Exploiting the independent channel property, $T_4 = MP_{\max}\mu_{SR}\mu_{RX}$. Further, substituting the value of θ_m, we can show that $T_5 = \sum_{m=1}^{M}\sum_{\substack{n=1\\ n\neq m}}^{M}\mathbb{E}\left[|h_{SR}^m|\,|h_{SR}^n|\,|h_{RX}^m|\,|h_{RX}^n|\right]\mathbb{E}\left[e^{j\left(\theta_{h_{RX}^m}-\theta_{h_{RX}^n}\right)}\right]$ $\times\,\mathbb{E}\left[e^{j\left(-\theta_{h_{RD}^m}+\theta_{h_{RD}^n}\right)}\right]$. Using [18, (10)] and observing that the PDF of the phase θ under Rician fading model is an even function of θ, the term $\mathbb{E}\left[e^{j\theta}\right]$ simplifies to $\zeta(K)$, which is defined in (10), where K is K_{RD} and K_{RX} for the RIS-D and RIS-X links, respectively. Finally, we can show that $T_5 = \frac{M(M-1)P_{\max}\pi^2\mu_{SR}\mu_{RX}}{16(K_{SR}+1)(K_{RX}+1)}\left[L_{1/2}\left(-K_{SR}\right)L_{1/2}\left(-K_{RX}\right)\zeta(K_{RX})\right.$ $\left.\times\zeta(K_{RD})\right]^2$. Combining all the necessary terms yields the final expression for I_{un} in (10).

C. Proof of Result 1

A selection rule that always chooses the zero transmit power option causes zero interference to X. It is, therefore, feasible for any I_{th}. Therefore, the set of all feasible selection rules is a non-empty set. Let ϕ be a feasible rule, for which $\mathbf{\Theta}$ be the RIS phase shift matrix and $P_S = P_{\max}$. For $\lambda > 0$, define an auxiliary function $\mathcal{L}_\phi(\lambda)$ associated with ϕ as follows:

$$
\mathcal{L}_\phi\left(\lambda\right) \triangleq \mathbb{E}\left[\text{SEP}\left(P_{\max},\mathbf{\Theta}\right) + \lambda I\left(P_{\max},\mathbf{\Theta}\right)\right]. \quad (14)
$$

Note that $\mathcal{L}_\phi(\lambda)$ is a function of both ϕ and λ.

Furthermore, we define $\overline{\phi}$ to be another feasible rule, which selects $\bar{P}_S = P_{\max}$ and $\bar{\mathbf{\Theta}}$ as follows:

$$
\bar{\mathbf{\Theta}} = \arg\min_{\mathbf{\Theta}}\left[\text{SEP}\left(P_{\max},\mathbf{\Theta}\right) + \lambda I\left(P_{\max},\mathbf{\Theta}\right)\right], \quad (15)
$$

where λ is chosen such that $\mathbb{E}\left[I\left(P_{\max},\bar{\mathbf{\Theta}}\right)\right] = I_{\text{th}}$. Thus, $\overline{\phi}$ is a feasible rule. We now prove that $\overline{\phi}$ is the optimal rule. From the definition of $\overline{\phi}$ in (15) it follows that $\mathcal{L}_{\overline{\phi}}\left(\lambda\right) \leq \mathcal{L}_\phi\left(\lambda\right)$. Therefore, using (14) and since $\mathbb{E}\left[I\left(P_{\max},\bar{\mathbf{\Theta}}\right)\right] = I_{\text{th}}$, we get

$$
\begin{aligned}
&\mathbb{E}\left[\text{SEP}\left(P_{\max},\bar{\mathbf{\Theta}}\right)\right] \\
&\quad\leq \mathbb{E}\left[\text{SEP}\left(P_{\max},\mathbf{\Theta}\right)\right] + \lambda\left(\mathbb{E}\left[I\left(P_{\max},\mathbf{\Theta}\right)\right] - I_{\text{th}}\right). \quad (16)
\end{aligned}
$$

Since ϕ is a feasible rule, $\mathbb{E}\left[I\left(P_{\max},\mathbf{\Theta}\right)\right] \leq I_{\text{th}}$. Thus, $\mathbb{E}\left[\text{SEP}\left(P_{\max},\bar{\mathbf{\Theta}}\right)\right] \leq \mathbb{E}\left[\text{SEP}\left(P_{\max},\mathbf{\Theta}\right)\right]$. Hence $\overline{\phi}$ yields the lowest average SEP among all feasible rules. Hence, it is the optimal rule.

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