Joint Transmit Antenna Selection and Passive Beamforming in IRS-Aided OTFS Systems

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***Abstract*—We develop a comprehensive and a practically relevant model for an intelligent reflecting surface (IRS)-assisted orthogonal time frequency space (OTFS) system in which the transmitter performs low complexity antenna selection (AS) and multiplexes data in the delay Doppler domain. We propose two new algorithms to jointly select antennas at the transmitter and program phase shifts at the IRS and also analyze their computational complexities. We show that the proposed algorithms cover a wide range of performance-complexity trade-off. Our work also suggests a trade-off between power-hungry radio frequency (RF) chains at the transmitter and power-efficient passive elements at the IRS while obtaining improved bit error rate (BER) performance. We also elucidate that 3-bit discrete phase shifts at the IRS can provide the same BER performance as an IRS with continuous phase shifts in an OTFS system.**

***Index Terms*—OTFS modulation, Intelligent Reflecting Surface (IRS), Antenna Selection (AS).**

I. Introduction

The sixth generation (6G) wireless systems must be designed to support high mobility applications. Under high mobility, the wireless channel behaves as doubly dispersive, since multipath propagation leads to interference among symbols and Doppler shift causes inter-symbol interference. The current systems based on orthogonal frequency division multiplexing (OFDM) are robust to inter-symbol interference. However reliability of data detection is adversely affected due to Doppler shift induced interference among subcarriers. Orthogonal time frequency space (OTFS) modulation that exploits the sparse and time-invariant nature of wireless channel in the delay-Doppler (DD) domain to send data, is a potential candidate in 6G to provide better resilience to high Doppler shifts [1], [2]. Intelligent reflecting surface (IRS) is being viewed as yet another potential 6G technology [3], [4]. It comprises of several low-cost scattering elements that do not require dedicated radio frequency (RF) chains. And by appropriately programming phase shifts induced by these passive elements, constructive superposition of signals reaching the receiver can be obtained, thereby maximizing received signal strength. IRS when integrated with OTFS can thus enhance data rates, energy efficiency and reliability of data detection in high Doppler applications foreseen in 6G systems.

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A. Related Prior Works on IRS-Aided OTFS systems

While in [5], an input-output relation was developed for fractional DD values and rectangular waveforms. The authors in [6] analyzed a multi-antenna OTFS system aided by an IRS using minimum mean square error (MMSE) detector. In [7], the authors developed a phase optimization method in which only the strongest DD channel response was accounted for, in designing phase shifts and analyzed error performance and data rates that can be achieved. The use of IRS-aided OTFS systems in space-air ground integrated networks was proposed and its performance with channel estimates was analyzed in [8].

B. Motivation and Our Contributions

We observe that there are limited works available on IRS-aided OTFS systems. Moreover, based on [5]–[8], we observe that most of the fairly recent works on IRS-aided OTFS systems considered transmitters where the number of RF chains are equal to number of transmit antennas. To reduce hardware complexity and power consumption, transmitters are generally equipped with fewer RF chains than the number of antennas and perform antenna selection (AS) [9], [10]. AS is a low complexity solution that retains the spatial diversity benefits of multi-antenna systems [9]. For this reason it is a part of several wireless standards [11].

Furthermore, to the best of our knowledge, there is no literature available on system modeling, optimization and performance analysis of IRS enabled OTFS systems where the transmitter is equipped with fewer RF chains than the number of antennas and multiplexes signal in the DD domain. In other words, beamformer design using the selected antennas at the transmitter and the phase shift design at the IRS for an IRS-aided OTFS system where the transmitter performs low complexity AS is an open problem and requires thorough investigation. To this end, we make the following key contributions:

- We develop a novel model for an IRS-aided OTFS system under transmit AS in which the transmitter has fewer RF chains than number of transmit antennas and is supported by an IRS to multiplex data in DD domain. While OTFS makes the system capable to operate in high Doppler scenarios arising due to high mobility or high carrier frequencies, the IRS reconfigures the propagation environment for better performance.
- We propose two novel algorithms for jointly selecting antennas at the transmitter and configuring phase shifts at the

IRS, namely, joint AS and exhaustive search (JAES) and joint AS and strongest delay Doppler channel response (JASD) based IRS phase programming and analyze their computational complexities. The two algorithms cover a wide range of performance-complexity trade-off. JASD is computationally less intensive with only marginal degradation in bit error rate (BER) compared to JAES.

- Through our numerical results, we infer that the power hungry RF chains at the transmitter can be traded with power efficient passive elements at IRS to obtain improved BER performance. In practice, an IRS cannot induce continuous phase shifts to the incident signal, our work shows that a 3 bit discrete phase shift at IRS may be enough to achieve performance identical to an IRS having continuous phase shift.

II. System Model

We consider an IRS-aided OTFS system in which the IRS consisting of L reflecting elements assists a transmitter with n_t transmit antennas and n_s ($\leq n_t$) RF chains in communicating with a receiver as shown in Figure 1. The direct path between the transmitter and the receiver is considered to be blocked. Let $x_t[k,l]$ denote the DD domain information symbol sent from the t^{th} transmit antenna in a two dimensional $M \times N$ bin represented by

$$\epsilon = \left\{ \left(\frac{k}{NT}, \frac{l}{M\Delta f} \right), k = 0, \ldots, N-1, l = 0, \ldots, M-1 \right\}, \tag{1}$$

where $\frac{1}{NT}$ and $\frac{1}{M\Delta f}$ capture the Doppler and delay resolutions, respectively. Also, N and M denote the total number of Doppler and delay taps, respectively. The channel impulse response from the t^{th} transmitter to the i^{th} element of IRS in DD domain is given by [5]

$$g_t^i(\tau,\nu) = \sum_{p=1}^{P} g_{p_t}^i \delta(\tau - \tau_{p_t}^i)\delta(\nu - \nu_{p_t}^i), \tag{2}$$

where $i = 1, \ldots, L$ and $t = 1, 2, \ldots, n_t$. In the equation above, P denotes the total number of paths from the t^{th} transmit antenna to the i^{th} element of IRS. Here $\tau_{p_t}^i$ and $\nu_{p_t}^i$ denote the delay and the Doppler shift, respectively, associated with path p and $g_{p_t}^i$ is the channel gain from t^{th} transmitter to i^{th} element along path p and we consider that $g_{p_t}^i \sim \mathcal{CN}(0, \frac{1}{P})$. The channel impulse response from the i^{th} IRS element to receiver can be expressed similarly in the DD domain as

$$h^i(\tau,\nu) = \sum_{q=1}^{Q} h_q^i \delta(\tau - \tau_q^i)\delta(\nu - \nu_q^i), \tag{3}$$

where Q denotes total number of paths from the i^{th} element of IRS to receiver, τ_q^i and ν_q^i denote the delay and the Doppler shift associated with path q and h_q^i denotes the channel gain from the i^{th} element to the receiver along path q and follows a $\mathcal{CN}(0, \frac{1}{Q})$ distribution. Let the reflection coefficient of i^{th} element be ϕ_i. Let γ_i and θ_i denote its amplitude and phase, respectively. Then, ϕ_i can be written as, $\phi_i = \gamma_i \ e^{j\theta_i}$. We assume that $\gamma_i = 1$, for all i. Let us define, $\nu_{p_t}^i + \nu_q^i = \nu_{pq_t}^i$, $\tau_{p_t}^i + \tau_q^i = \tau_{pq_t}^i$, $\rho_{pq_t}^i = e^{-j2\pi\nu_q^i\tau_{p_t}^i}$ and $g_{p_t}^i \times h_q^i \times \rho_{pq_t}^i = h_{pq_t}^i$. Then, the DD domain symbol $y_t^i[k,l]$ at the receiver coming from the t^{th} transmit antenna via the i^{th} IRS element is given by [5]

$$y_t^i[k,l] = e^{j\theta_i} \sum_{q=1}^{Q}\sum_{p=1}^{P} h_{pq_t}^i e^{-j2\pi\nu_{pq_t}^i\tau_{pq_t}^i} \times x_t[[k - \beta_{pq_t}^i]_N, [l - \alpha_{pq_t}^i]_M], \tag{4}$$

where $\alpha_{pq_t}^i$ and $\beta_{pq_t}^i$ are assumed to be integral multiple of delay and Doppler resolutions respectively. Now, (4) can be expressed in a vectorized form as $\boldsymbol{y_t^i} = \phi_i \boldsymbol{H_t^i x_t}$, where $\boldsymbol{y_t^i} \in C^{MN\times 1}$, $\boldsymbol{x_t} \in C^{MN\times 1}$ and $\boldsymbol{H_t^i} \in C^{MN\times MN}$ represents the cascaded channel-matrix from t^{th} transmitter to receiver through i^{th} element of the IRS. Summing up the signals coming from all the L elements at the receiver, we get DD domain input-output relation as

$$\boldsymbol{y_t} = \sum_{i=1}^{L} \phi_i \boldsymbol{H_t^i x_t} + \boldsymbol{v_t} = \boldsymbol{H_t x_t} + \boldsymbol{v_t}, \tag{5}$$

where $\boldsymbol{y_t} \in \mathcal{C}^{MN\times 1}$ is the combined received vector from all the elements corresponding to the t^{th} transmit antenna and $\boldsymbol{v}_t \in \mathcal{C}^{MN\times 1}$ denotes the noise vector at the receiver. Thus, the input-output relation of an IRS-aided OTFS system with n_t antennas at the transmitter is given by, $\boldsymbol{y'} = \boldsymbol{H'x'} + \boldsymbol{v'}$, where $\boldsymbol{y'} \in \mathbb{C}^{MN\times 1}$ is the received signal vector and $\boldsymbol{H'} = [\boldsymbol{H}_1 \ \boldsymbol{H}_2 \ \ldots \ \boldsymbol{H}_{n_t}] \in \mathbb{C}^{MN\times n_t MN}$ is the overall channel matrix with $\boldsymbol{H}_j$ being the $MN \times MN$ equivalent channel matrix between the j^{th} transmit antenna and receive antenna. The transmit vector is represented by $\boldsymbol{x'} = [\boldsymbol{x}_1 \ \boldsymbol{x}_2 \ \ldots \ \boldsymbol{x}_{n_t}]^T \in \mathbb{C}^{n_t MN\times 1}$ and $\boldsymbol{v'} \in \mathbb{C}^{MN\times 1}$ represents noise. Assuming availability of channel state information at the transmitter, let

$$\boldsymbol{x'} = \underbrace{\left[\frac{\boldsymbol{H}_1^H}{||\boldsymbol{H}_1||} \ \ \frac{\boldsymbol{H}_2^H}{||\boldsymbol{H}_2||} \ \ \cdots \ \ \frac{\boldsymbol{H}_{n_t}^H}{||\boldsymbol{H}_{n_t}||} \right]^T}_{\boldsymbol{H}_b^T} \boldsymbol{x}. \tag{6}$$

Then, the input output relation reduces to

$$\boldsymbol{y'} = \underbrace{\boldsymbol{H'H}_b^T}_{\triangleq \boldsymbol{H}_e} \boldsymbol{x} + \boldsymbol{v}, \tag{7}$$

where the effective channel matrix, $\boldsymbol{H}_e = \boldsymbol{H'H}_b^T = \sum_{t=1}^{n_t} \frac{\boldsymbol{H}_t \boldsymbol{H}_t^H}{||\boldsymbol{H}_t||}$. Under transmit AS the transmitter connects the n_s best antennas among the n_t antennas to the available RF chains and the effective channel matrix is given by

$$\widetilde{\boldsymbol{H}} = \sum_{t=t_1}^{t_{n_s}} \frac{\boldsymbol{H}_t \boldsymbol{H}_t^H}{||\boldsymbol{H}_t||}, \tag{8}$$

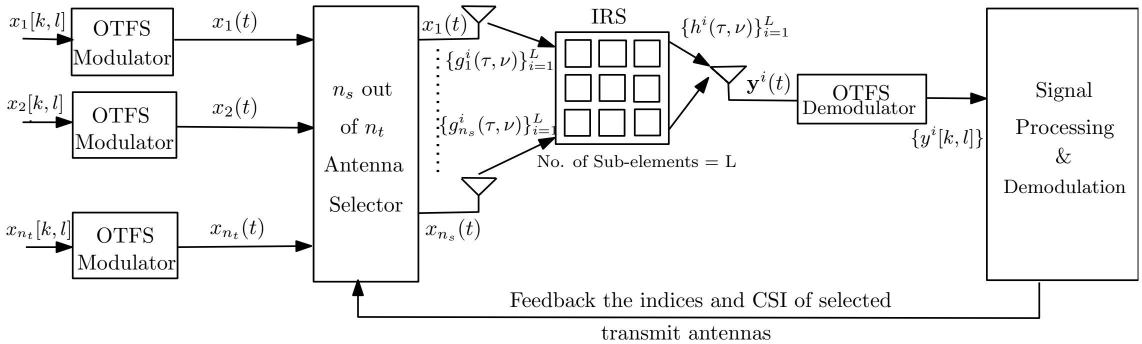

Fig. 1: System Model for Mutiple Input Single Output IRS-Aided OTFS under Transmit AS

where t_1, t_2,..., t_{n_s} denote the indices of the n_s antennas that get selected for transmission. At the receiver, MMSE detector is employed and the detected symbol is given by

$$\tilde{\boldsymbol{x}} = (\widetilde{\boldsymbol{H}}^H \widetilde{\boldsymbol{H}} + \sigma^2 \boldsymbol{I}_{MN})^{-1} \widetilde{\boldsymbol{H}}^H \boldsymbol{y}, \tag{9}$$

where $\sigma^2 \boldsymbol{I}_{MN}$ is covariance matrix of the noise vector.

III. Joint AS and IRS Phase Optimization Algorithms

The main challenge involved in transmit AS in an IRS-aided OTFS system is to select the best set of antennas while optimizing the phase shifts induced by IRS elements so that the Forbenius norm of the overall effective channel matrix gets maximized. In this section, we propose two novel AS and IRS phase optimization algorithms and analyze their computational complexity. In these two algorithms, $[j]$ = max($\boldsymbol{b}$) returns the index j of the largest element in the vector $\boldsymbol{b}$.

A. JAES Based IRS Programming

We note that in order to select n_s best antennas out of n_t antennas, there are ${}^{n_t}C_{n_s}$ antenna subsets or number of antenna combinations possible. In this method, for each combination or each subset of antennas, we generate a large number of L-length random phase vectors where each element of the vector is uniformly distributed in $[-\pi, \pi]$. Let us assume, the total number of such randomly generated phase vectors is U. The u^{th} realization of the phase shift vector for the c^{th} antenna subset is denoted by $\boldsymbol{\Theta}_u^c$ = $[\theta_{1u}^c, \theta_{2u}^c, ..., \theta_{Lu}^c]$. For each possible subset, we find out the corresponding optimal phase vectors from the randomly generated phase vectors. To be precise, for the c^{th} subset, the channel matrix from the t^{th} transmit antenna in the u^{th} realization is given by

$$\boldsymbol{H}_t(c,u) = \sum_{i=1}^{L} e^{j\theta_{iu}^c} \boldsymbol{H}_t^i. \tag{10}$$

Next we select that particular phase vector among the U realizations to be the optimum one which gives us maximum Forbenius norm of effective channel matrix. Mathematically, the selected phase shift configuration u^* is given by

Algorithm 1: JAES Based IRS Programming

Input: $\boldsymbol{H}_t^i$ where i = 1,2,...,L, k = 1,2,...,n_t and U random realizations of the L-length phase shift vectors.
Output: $\{t_1, t_2, \ldots, t_{n_s}\}$ and $\boldsymbol{\Theta}^*$.
for *each* ${}^{n_t}C_{n_s}$ *possible combinations of* $(t_1, t_2, \ldots, t_{n_s})$ **do**
  Generate: $\boldsymbol{\Theta}_u^c$= $[\theta_{1u}^c \;\; \theta_{2u}^c \;\; \ldots \;\; \theta_{Lu}^c]$ where $\theta_{iu}^c \in \mathcal{U}[-\pi, \pi]$; i= 1, 2,..., L; c = 1,2,...,${}^{n_t}C_{n_s}$.
  Initialize $u^* = 0$ and $\psi_c^* = 0$;
  for $u = 1$,$u \leq U$,$u++$ **do**
    $\boldsymbol{H}_t \leftarrow \sum_{i=1}^{L} e^{j\theta_{iu}^c} \boldsymbol{H}_t^i$; where $t = t_1^c, t_2^c, \ldots, t_{n_s}^c$
    $\widetilde{\boldsymbol{H}}$ = $\sum_{t=t_1^l}^{t_{n_s}^l} \frac{\boldsymbol{H}_t \boldsymbol{H}_t^H}{||\boldsymbol{H}_t||}$;
    $\boldsymbol{\psi}^c[u] \leftarrow ||\widetilde{\boldsymbol{H}}||^2$;
  u^* = max($\boldsymbol{\psi}^c$);
  $\tilde{\boldsymbol{\psi}}[c] = \boldsymbol{\psi}^c[u^*]$;
c^* = max ($\tilde{\boldsymbol{\psi}}$);
return c^* and $\boldsymbol{\Theta}_{u^*}^{c^*}$

$$u^* = \underset{u}{\text{argmax}} \left\| \sum_{t=t_1^c}^{t_{n_s}^c} \frac{\boldsymbol{H}_t(c,u) \boldsymbol{H}_t^H(c,u)}{||\boldsymbol{H}_t(c,u)||} \right\|^2, \tag{11}$$

where $t_1^c, t_2^c, \ldots, t_{n_s}^c$ denote the indices of the antennas belonging to the c^{th} subset and $u \in \{1, 2, \ldots, U\}$. Likewise, we determine the optimal phase shift vectors for each of the ${}^{n_t}C_{n_s}$ antenna subsets. So far, we have found the optimal phase configuration for each subset. Thereafter, we select that subset and the corresponding phase pattern for which the effective channel strength is maximized. To be specific, the best antenna combination c^* is given by

$$c^* = \underset{c}{\text{argmax}} \left\| \sum_{t=t_1^c}^{t_{n_s}^c} \frac{\boldsymbol{H}_t^*(c) \; \boldsymbol{H^*}_t^H(c)}{||\boldsymbol{H}_t^*(c)||} \right\|^2, \tag{12}$$

where $c \in \{1, 2, \ldots, {}^{n_t}C_{n_s}\}$, $\boldsymbol{H}_t^* = \sum_{i=1}^{L} e^{j\theta_{iu^*}^c} \boldsymbol{H}_t^i$ and the corresponding IRS phase shift vector is $\boldsymbol{\Theta}^*$ = $\boldsymbol{\Theta}_{u^*}^{c^*}$. The above procedure is summarized in Algorithm 1. The computational complexity of JAES depends on the complexity of max$(\cdot)$ function that has been used in the algorithm. The complexity associated with finding the indices of the largest element in any N length vector using max function, that we have used, is $\mathcal{O}(N)$. Time complexity calculation for JAES can be explained based on the following steps:

- For each randomly generated phase vector for each antenna subset, computation of $\boldsymbol{H}_t$ involves L complex additions. So, the time complexity is given by $\mathcal{O}({}^{n_t}C_{n_s}UL)$.
- For each randomly generated phase vector for each antenna subset, computation of $\widetilde{\boldsymbol{H}}$ includes n_s matrix additions and multiplications. So, the time complexity is given by $\mathcal{O}({}^{n_t}C_{n_s}Un_s)$.
- For each antenna combination, to get the optimal phase vector, we need to find u^* by taking max of $\boldsymbol{\psi^c}$, which is an U length vector. So, the time complexity becomes $\mathcal{O}({}^{n_t}C_{n_s}U)$.
- Similarly, to find the optimum antenna subset or combination c^* we take the max of $\widetilde{\boldsymbol{\psi}}$ which is a ${}^{n_t}C_{n_s}$ length vector. So, the time complexity becomes $\mathcal{O}({}^{n_t}C_{n_s})$.

Hence, the overall complexity for JAES will be the sum of the time complexities of all these steps. It is clear from the above calculation that the complexity of this technique depends on the number of random phase vectors over which the exhaustive search is carried out and scales as a function of U. To this end, we next propose a novel low complexity JASD based IRS programming [7], in order to reduce time-complexity associated with AS and IRS programming.

B. JASD Based IRS Programming

In the DD domain, there are P paths between any antenna at the transmitter and any element at the IRS and Q paths between any element at the IRS and the receiver. Among the PQ available paths, JASD based IRS programming uses the path along which the delay Doppler response of the cascaded channel via the IRS is the strongest to program phase shifts at IRS and select antennas at transmitter. To be specific, if we let $h^i_{\tilde{p}_t}$ denote the cascaded channel coefficient through i^{th} IRS element corresponding to the t^{th} transmit antenna where $\tilde{p} \in \{1, 2, \ldots, PQ\}$. As discussed before, there are ${}^{n_t}C_{n_s}$ antenna subsets or combinations possible. Therefore, for the c^{th} antenna subset, the strongest path is given by

$$(\tilde{p}_c)^* = \underset{\tilde{p}_c \in \{1,2,\ldots,(PQ)\}}{\text{argmax}} \left(\sum_{t=t^c_1}^{t^c_{n_s}} \sum_{i=1}^{L} |h^i_{\tilde{p}_t}|^2 \right). \tag{13}$$

Furthermore, in this method, to align the phase of the IRS reflected beam from the t^{th} antenna, the phase of the i^{th} IRS element is configured as [7]

$$\theta^{\Upsilon}_i = -\angle h^i_{\tilde{p}^*_t}, \tag{14}$$

where $\angle h^i_{\tilde{p}^*_t}$ is the phase of the path that has the strongest DD channel response via the i^{th} IRS element and Υ is the index of the antenna belonging to the c^{th} antenna subset. Let the corresponding phase shift vector be $\boldsymbol{\Theta}^{\Upsilon}$. Now, for all of these antennas that belong to a subset, there has to be a common phase shift coefficient. Therefore, there is need to determine the optimal phase shift vector from all of these phase vectors that we get for each antenna in the subset. For that, we propose two techniques:

1) *MAX Phase Shift Method :* We start by taking the phase shift vector associated with one of the antennas in the subset. Using that, we determine the effective channel matrix
$$\boldsymbol{H}_t(\boldsymbol{\Theta}^{t^c_1}) = \sum_{i=1}^{L} e^{j\theta_i^{t^c_1}} \boldsymbol{H}^i_t, \tag{15}$$
where $t \in \{t^c_1, t^c_2, \ldots, t^c_{n_s}\}$. We then compute the effective channel matrix for each phase shift pattern belonging to all other antennas in the subset. The phase vector that provides the highest Forbenius norm of the effective channel matrix is chosen, i.e,
$$\boldsymbol{\Theta}^c = \underset{\Upsilon \in \{t^c_1, t^c_2, \ldots, t^c_{n_s}\}}{\text{argmax}} \left|\left| \sum_{t=t^c_1}^{t^c_{n_s}} \frac{\boldsymbol{H}_t(\boldsymbol{\Theta}^{\Upsilon})\boldsymbol{H}^H_t(\boldsymbol{\Theta}^{\Upsilon})}{||\boldsymbol{H}_t(\boldsymbol{\Theta}^{\Upsilon})||} \right|\right|^2 \tag{16}$$
2) *MEAN Phase Shift Method:* The other strategy that we propose is to use the mean of the phase shift vectors corresponding to antennas in the subset as the phase shift coefficient for the subset. Mathematically,
$$\theta^c_i = \frac{1}{n_s} \sum_{t=t^c_1}^{t^c_{n_s}} \theta^t_i, \quad \text{for } 1 \le i \le L. \tag{17}$$

The computational complexity of JASD based IRS programming can be calculated as follows :

- For each of the existing paths $\tilde{p}$ for each of ${}^{n_t}C_{n_s}$ antenna subsets, calculating $\boldsymbol{h_c}$ needs n_sL number of additions. Hence, the corresponding time complexity scales as $\mathcal{O}({}^{n_t}C_{n_s}PQn_sL)$.
- For each of ${}^{n_t}C_{n_s}$ antenna subsets, to get the strongest path, we need to feed a PQ length vector $\boldsymbol{h_c}$ to max function. So, the time complexity for that becomes $\mathcal{O}({}^{n_t}C_{n_s}PQ)$.
- With MAX phase shift method, to get $\boldsymbol{H^c_t}$ for each of the phase vectors corresponding the each of the antennas in the combination, the time complexity is $\mathcal{O}({}^{n_t}C_{n_s}n_sL)$ + $\mathcal{O}({}^{n_t}C_{n_s}n_s^2)$. And to get the optimal phase shift vector, we apply max function to n_s length vector $\boldsymbol{\psi}$. So, the corresponding time complexity associated with max operation scales as $\mathcal{O}({}^{n_t}C_{n_s}n_s)$.
- With MEAN phase shift method, we get the optimal phase vector with n_s additions of corresponding angles and then dividing them up by n_s. So, the complexity is $\mathcal{O}({}^{n_t}C_{n_s}n_s)$.
- With optimal phase shift, we calculate $\boldsymbol{H}_t$ and $\boldsymbol{H}$ with complexity $\mathcal{O}({}^{n_t}C_{n_s}L)$ + $\mathcal{O}({}^{n_t}C_{n_s}n_s)$.

Algorithm 2: JASD Based IRS Programming

Input: $h^{i}_{\tilde{p}_t}$ and $\boldsymbol{H^i_t}$, where $i = 1, 2, \ldots, L$, $T = 1, 2, \ldots, n_t$, $\tilde{p} = 1, 2, \ldots, PQ$.
Output: $\{t_1, t_2, \ldots, t_{n_s}\}$ and $\boldsymbol{\Theta^*}$
for *each* $({}^{n_t}C_{n_s})$ *possible combinations of* $(t_1, t_2, \ldots, t_{n_s})$ **do**
  Initialize $\tilde{p}^*$ = 0 and h^* =0;
  for $\tilde{p}$=*1*,$\tilde{p} \leq PQ$, $\tilde{p}$ *++* **do**
    $\boldsymbol{h_c}[\tilde{p}] \leftarrow \sum_{t=t^c_1}^{t^c_{n_s}} \sum_{i=1}^{L} |h^i_{\tilde{p}_t}|^2$; where $c = 1, 2, \ldots, {}^{n_t}C_{n_s}$.
  $\tilde{p}^*$ = max($\boldsymbol{h_c}$);
  $\theta^{\Upsilon}_i = -\angle h^i_{\tilde{p}^* T}$; where $\Upsilon \in \{t^c_1, t^c_2, \ldots, t^c_{n_s}\}$
  CASE 1: MAX Phase Shift Method
  for n = *1*,$n \leq n_s$, n*++* **do**
    $\boldsymbol{H}^c_t \leftarrow \sum_{i=1}^{L} e^{j\theta_i^{t^c_n}} \boldsymbol{H}^i_t$; where $t \in \{t^c_1, t^c_2, \ldots, t^c_{n_s}\}$
    $\boldsymbol{H^c}$= $\sum_{t=t^c_1}^{t^c_{n_s}} \frac{\boldsymbol{H}^c_t \boldsymbol{H}^{cH}_t}{||\boldsymbol{H}^c_t||}$;
    $\boldsymbol{\psi}(n)$ = $||\boldsymbol{H}^c||^2$;
  n^* = max($\boldsymbol{\psi}$);
  θ^*_i = $\theta_i^{t^c_{n^*}}$;
  CASE 2: MEAN Phase Shift Method
  θ^*_i = $\frac{1}{n_s} \sum_{t=t^c_1}^{t^c_{n_s}} \theta^t_i$;
  $\boldsymbol{\Theta^*_c}$ = $[\theta^*_1 \theta^*_2 \ldots \theta^*_L]$;
  $\boldsymbol{H_t} \leftarrow \sum_{i=1}^{L} e^{j\theta^*_i} \boldsymbol{H^i_t}$; where $t \in \{t^c_1, t^c_2, \ldots, t^c_{n_s}\}$;
  $\boldsymbol{H}$= $\sum_{t=t^c_1}^{t^c_{n_s}} \frac{\boldsymbol{H}_t \boldsymbol{H}^H_t}{||\boldsymbol{H}_t||}$;
  $\tilde{\boldsymbol{\psi}}[c] \leftarrow ||\boldsymbol{H}||^2$;
c_m = max $(\tilde{\boldsymbol{\psi}})$;
return $(c_m)^{\text{th}}$ $\{t_1, t_2, \ldots t_{n_s}\}$ pair and corresponding $\boldsymbol{\Theta^*}$ = $\boldsymbol{\Theta^*_{c_m}}$

- To find the optimal combination, we have an ${}^{n_t}C_{n_s} n_s$ length vector fed to max function. So, the time complexity is $\mathcal{O}({}^{n_t}C_{n_s})$.

We note that JASD is less complex when compared to JAES, since in practice the total number of paths PQ is bounded by the total number of delay and Doppler taps and it will be significantly lower than U. Furthermore, we note that MEAN phase shift method is lower in complexity compared to MAX phase shift method.

IV. Numerical Results

In this section, we present our numerical results to illustrate the BER performance of the proposed algorithms. We elucidate the effect of the number of IRS elements, number of RF chains and number of antennas at the transmitter and discrete phase shifts on BER. The system parameters are listed in Table I. Fig. 2 plots BER as a function of SNR for different

TABLE I: Simulation parameters [5]

Frame Size (M, N)	(2,2)
Carrier Frequency (f_c)	4 GHz
Sub-carrier Spacing (Δf)	3.45 kHz
DD (τ, ν) for 2 paths	$(0, 0)$, $\left(\frac{1}{M\Delta f}, \frac{1}{NT}\right)$
Maximum Speed	506.25 km/hr
Maximum Doppler Spread	1.875 kHz
Modulation	QAM-4
Detection Scheme	MMSE

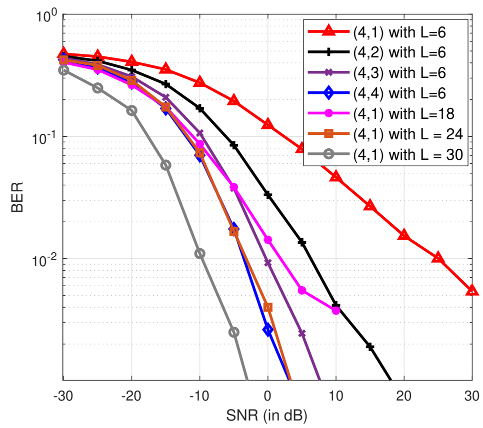

Fig. 2: JAES: Impact of number of RF chains and IRS elements on BER $(U = 10^4)$

combinations of (n_t, n_s) and L for JAES based IRS phase programming. Note that (n_t, n_s) refers to a configuration where the source has n_t transmit antennas and n_s RF chains. Also, 10^4 number of channel realizations were generated in this Monte Carlo simulation. We observe that keeping L fixed at 6 and n_t fixed at 4, the BER improves as we increase the number of active RF chains from 1 to 4. This is because an increase in the number of RF chains leads to a boost in the receive SNR due to active beamforming-assisted coherent combining of signals coming from the selected antennas. Another interesting observation that we make is that keeping n_t fixed at 4 and n_s fixed at 1, the BER performance improves as L increases. This is due to constructive interference aided boost in the receive SNR obtained from passive beamforming at IRS. In fact, $(4, 1)$ with $L = 30$ gives better BER performance than $(4, 4)$ with $L = 6$. In other words, RF chains at the source are power-hungry, bulky and costly. It can be traded with IRS elements to obtain improved BER.

Fig. 3 plots the BER as a function of SNR for a (4,2) configuration with $L = 6$ for both JAES and JASD based IRS programming. We observe that there is a trade-off between performance and complexity. To be specific, JASD gives slightly poorer BER compared to JAES at relatively lower complexity. Furthermore, the MEAN method gives marginally poor BER compared to the MAX method which has higher complexity.

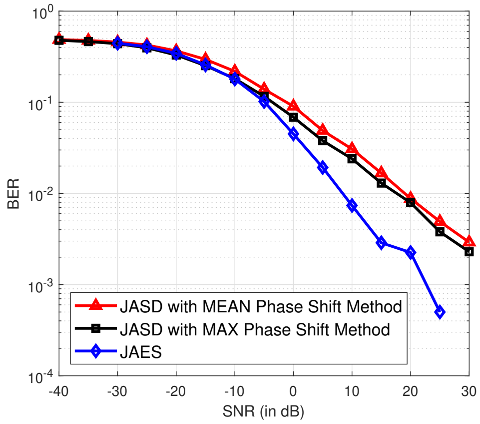

Fig. 3: BER performance for a ($n_t = 4$, $n_s = 2$, $L = 6$) IRS-aided OTFS system with the proposed algorithms

In practice, the IRS elements are not capable of inducing continuous phase shifts on the incident signal. In general, with q pin diodes fabricated on every element, 2^q levels or 2^{q-1} bit phase shift can be obtained. In Figure 4, we plot BER as a function of SNR for 1-bit, 2-bits, 3-bits and infinite precision IRS (∞-bits) for JAES based AS and IRS programming. We observe that as the number of bits increases, the BER performance of IRS-aided OTFS systems improves due to better coherent addition of reflected signals received at the user. In fact, we observe that it suffices to have 3-bit phase shift at IRS to obtain performance identical to infinite precision IRS. Similar trends were also observed for JASD based IRS programming and are not shown to avoid clutter and due to space constraints.

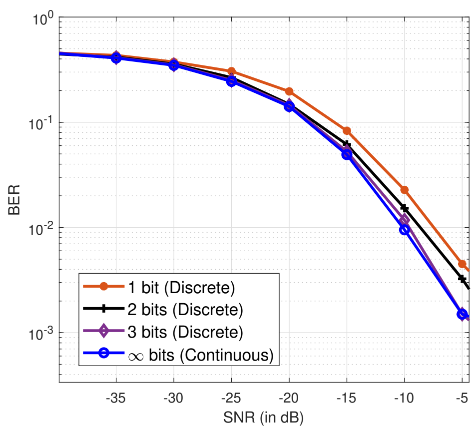

Fig. 4: JAES: Impact of discrete phase shifts on BER ($n_t = 4$, $n_s = 2$, $L = 24$, $U = 10^4$)

V. Conclusions

We considered an IRS-aided OTFS system under transmit AS in which the transmitter has fewer RF chains than the number of transmit antennas. For joint AS and IRS phase programming, we proposed two new algorithms, namely, JAES and JASD. The performance-complexity trade-offs covered by the two methods span a large spectrum. When compared to JAES, JASD has a marginal BER deterioration and is computationally less complex. In order to achieve better BER performance, our work also suggested a trade-off between power-hungry RF chains at source and power-efficient passive elements at IRS. We also demonstrated that a 3 bit phase shift at the IRS may be sufficient to provide the same performance as an IRS with continuous phase shift. Generalizations to practical pulse shaping, fractional Doppler performance with channel estimates and multiple receive antennas are some interesting avenues for future research.

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