A NOVEL DEMODULATION AND SELECTION PILOT POWER TRADE-OFF FOR CODEBOOK-BASED IRS WITH IMPERFECT CHANNEL ESTIMATES

Sriram Ganesan† *Neelesh B. Mehta*† *Rimalapudi Sarvendranath*⋆

† ECE dept., Indian Institute of Science, Bangalore, India
⋆ EE dept., Indian Institute of Technology, Tirupati, India

ABSTRACT

The codebook-based scheme for intelligent reflecting surfaces (IRSs) provides flexibility in controlling the training overhead. In it, the reflection pattern with the largest received signal strength is selected from a pre-specified codebook and configured at the IRS. We analyze a training scheme that exploits a novel trade-off between the powers allocated for selection pilots, which are used to select the reflection pattern, and the demodulation pilot, which is used to estimate the channel for demodulation. We develop a novel selection-aware estimator of the beamforming gain of the selected reflection pattern. We derive a tight bound for the achievable rate and an elegant closed-form expression for the beamforming gain. These account for the impact of imperfect channel estimates on the selection of the reflection pattern and the coherent demodulation of the data symbols. The proposed scheme achieves a higher rate than conventional schemes by allocating substantially different powers to the selection and demodulation pilots and data symbols.

***Index Terms*—** Intelligent reflecting surface, estimation, training, beamforming, power adaptation.

1. INTRODUCTION

An intelligent reflecting surface (IRS) consists of many small, low-cost passive elements, each of which can induce a controlled phase shift to the incident electromagnetic signal. This controllability can be exploited to improve throughput, coverage, and energy efficiency of next-generation wireless communication systems [1, 2].

The transmitter or access point (AP) requires the estimates of the cascaded channels from it to the user through each IRS element to determine the optimal reflection pattern at the IRS [1,3,4]. Hence, cascaded channel estimation plays a crucial role in an IRS-aided system and has spurred a lot of research. In the classical on-off estimation scheme [5, 6], the IRS turns on one element at a time. However, as many pilots as the IRS elements are needed. The discrete Fourier transform (DFT)-based scheme [3, 7], in which different columns of the DFT matrix are used as reflection patterns for different pilot symbols, achieves a lower mean square error (MSE) than the on-off scheme, but with the same training overhead. For IRSs with tens to hundreds of elements, the training overhead can be large. To reduce this overhead, IRS element grouping [3], active IRS elements [8], and different machine-learning approaches [9–11] have been pursued. However, these works do not consider pilot power adaptation.

An alternative to the above estimation-based schemes is the codebook-based scheme [12–14]. In it, K pre-designed reflection patterns are used at the IRS, one for each pilot transmission. The pattern that maximizes the received signal strength is selected and configured at the IRS during data transmission. The codebook-based scheme is practically appealing because its pilot overhead is fixed. Its control signaling overhead, which the AP incurs to send the index of the selected reflection pattern, is also significantly lower than that of the estimation-based schemes, which need to communicate the phase shift of each IRS element.

Imperfect channel estimates, which are inevitable in practice because of finite pilot powers, affect the codebook-based scheme in two ways: i) they can cause a sub-optimal reflection pattern to be selected, and ii) this sub-optimal pattern together with imperfect channel estimates affect demodulation and lower the data rate. Only a subset of these effects are studied in the fixed pilot power schemes in the literature on codebook-based training [12, 15–19].

1.1. Focus and Contributions

We analyze a codebook-based IRS-assisted communication system. We account for imperfect channel estimates during the selection of the reflection pattern and coherent demodulation. We leverage the intuition that the former is more robust to the imperfect estimates. This leads us to a novel trade-off between the powers allocated for selection and demodulation, which we exploit to improve the rate.

We make the following contributions:

i) We propose a novel training scheme that separates the tasks of selection of the reflection pattern and demodulation. In it, the AP first sends K selection pilots to the user, one for each reflection pattern configured at the IRS. The user selects the reflection pattern with the highest received signal strength and feeds back its index to the AP, which the AP forwards to the IRS. The AP then sends a demodulation pilot to enable the user to estimate the effective channel gain, followed by data. This scheme does not require channel reciprocity.

ii) For the above training scheme, we analyze the achievable rate for orthogonal codebooks, which includes the DFT and Hadamard matrix based codebooks [7]. The analysis accounts for the impact of imperfect channel estimates on the selection of the reflection pattern and demodulation, and the training, control signaling, and feedback overheads. It relies upon two novel theoretical results. First, we derive in closed form the probability density function (PDF) of the effective channel gain corresponding to the selected reflection pattern and its beamforming gain. Second, we derive a novel selection-aware linear minimum mean square error (LMMSE) estimator of the effective channel gain, which is used for coherent demodulation. It exploits the fact that the statistics of the effective channel gain of the selected pattern differ from those of an arbitrary pattern.

iii) Our numerical results show that the proposed scheme achieves a higher rate than the conventional codebook-based and estimation-based schemes in regimes of practical interest. The

This work was supported in part by the "Next Generation Wireless Research and Standardization on 5G and Beyond" project funded by the Ministry of Electronics and Information Technology, Govt. of India. The work of S. Rimalapudi was supported by a DST INSPIRE research grant.

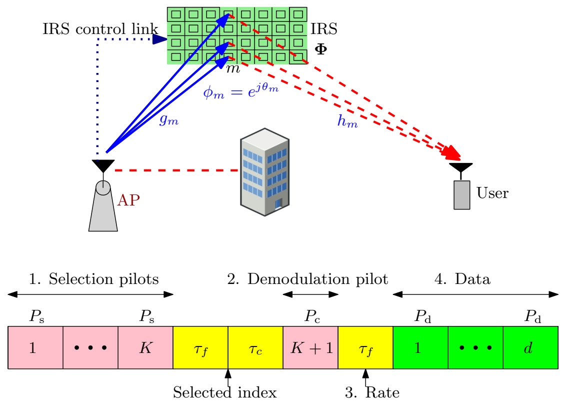


Fig. 1. System model and an illustration of the codebook-based training and data transmission scheme.

reason behind this is its ability to allocate higher power to the demodulation pilot than a selection pilot. This advantage has not been exploited in the literature [12–19].

1.2. Outline and Notations

The system model and the proposed transmission scheme are described in Section 2. Section 3 derives an achievable rate expression and optimizes it. Numerical results are presented in Section 4. Our conclusions follow in Section 5.

Notation: We denote the probability of an event B by $\Pr(B)$ and the expectation of a random variable (RV) X by $\mathbb{E}[X]$. The notation $X \sim \mathcal{CN}(\mu, \sigma^2)$ means that X is a complex normal RV with mean μ and variance σ^2.

2. SYSTEM MODEL AND PROPOSED SCHEME

The system model is illustrated in Figure 1. A single antenna AP transmits data to a single antenna user using an IRS with M elements. The direct link between the AP and the user is blocked due to obstacles [1, 14]. Let g_m and h_m denote the complex channel gains from the AP to the m^{th} IRS element and from m^{th} IRS element to the user, respectively. We consider quasi-static flat fading channel model. Let $\mathbf{g} = [g_1, g_2, \ldots, g_M]^T$ and $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$, where $(.)^T$ denotes transpose.

For analytical tractability, we focus on the following model: i) $h_1, h_2, \ldots, h_M$ are independent [4, 6, 12]. ii) A dominant line-of-sight (LoS) link exists between the AP and the IRS [12]. This is justifiable when the AP is deployed on a tower and the IRS on a rooftop. Hence, $\mathbf{g}$ is a deterministic LoS channel gain vector. iii) The IRS-user link undergo Rayleigh fading with path-loss β_h. Therefore, $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \beta_h \mathbf{I}_M)$. Let β_g denote the path-loss of the AP-IRS link.

We note that the achievable rate of codebook-based training has not been analyzed even for the above model. While multiple antennas have been considered at the transmitter in [12, 14–18], perfect channel estimates are implicitly assumed for demodulation and in the capacity expressions. Instead, in [3], the focus is on MSE and the impact of imperfect estimates on demodulation is not considered.

When AP transmits symbol x and the reflection pattern $\boldsymbol{\phi} = [\phi_1, \phi_2, \ldots, \phi_M]^T$ is used, the signal y received by the user is [1]

$$y = \left(\sum_{m=1}^{M} h_m \phi_m g_m \right) x + n = \mathbf{r}^T \boldsymbol{\phi} x + n, \tag{1}$$

where $\mathbf{r} = [r_1, r_2, \ldots, r_M]^T = [h_1 g_1, h_2 g_2, \ldots, h_M g_M]$ is the cascaded reflected channel gain vector and $n \sim \mathcal{CN}(0, \sigma^2)$ is additive white Gaussian noise.

2.1. Proposed Training and Transmission Scheme

A codebook consisting of $K \leq M$ orthogonal reflection patterns $\boldsymbol{\phi}(1), \boldsymbol{\phi}(2), \ldots, \boldsymbol{\phi}(K)$ is used for generating the reflection coefficients at the IRS. Our scheme has following four phases:

i) Selection Phase: The AP sequentially transmits K selection pilots, each with power P_s. When the i^{th} pilot $p = 1$ is transmitted, the reflection pattern at the IRS is $\boldsymbol{\phi}(i)$. The signal y_i at the user is

$$y_i = \sqrt{P_s} \mathbf{r}^T \boldsymbol{\phi}(i) p + n_i = \sqrt{P_s} h_{\text{eq}}(i) p + n_i. \tag{2}$$

Here, $n_1, n_2, \ldots, n_K \sim \mathcal{CN}(0, \sigma^2)$ are independent and identically distributed (i.i.d.) additive white Gaussian noises, and $h_{\text{eq}}(i) = \mathbf{r}^T \boldsymbol{\phi}(i)$ is the effective end-to-end AP-user channel.

The user selects the reflection pattern index denoted by s as

$$s = \arg\max_{i \in \{1,2,\ldots,K\}} \{|y_i|^2\}, \tag{3}$$

where $|.|$ denotes the absolute value. The index s is fed back to the AP using only $\lceil \log_2(K) \rceil$ bits, where $\lceil . \rceil$ denotes the ceiling function. The AP then sends s to the IRS via the control link, again using $\lceil \log_2(K) \rceil$ bits. Let τ_f and τ_c denote the feedback and control signaling durations, respectively.

ii) Estimation Phase: $\boldsymbol{\phi}(s)$ is configured as the reflection pattern at the IRS. The AP sends an extra demodulation pilot $p = 1$ with power P_c to enable the user to accurately estimate the selected downlink channel gain $h_{\text{eq}}(s)$ for coherent demodulation. The signal y_c at the user is

$$y_c = \sqrt{P_c} h_{\text{eq}}(s) + n_c, \tag{4}$$

where $n_c \sim \mathcal{CN}(0, \sigma^2)$ is additive white Gaussian noise.

iii) Feedback of Rate: Based on its estimate of $h_{\text{eq}}(s)$, the user feeds back to the AP a data rate R that it can receive. We derive the expression for R below.

iv) Data Transmission Phase: The AP sends d downlink data symbols to the user with power P_d and rate R.

Note that P_s, P_c, and P_d can be different.

3. PROBLEM FORMULATION AND ANALYSIS

We first develop a novel selection-aware LMMSE estimate for $h_{\text{eq}}(s)$ and then derive an upper bound on the achievable rate. We then formulate our optimal pilots and data power allocation problem.

3.1. Selection-Aware Estimator for $h_{\text{eq}}(s)$

We employ the LMMSE estimator for estimating $h_{\text{eq}}(s)$ from y_c. The estimate $\hat{h}_{\text{eq}}(s)$ is given by [20, Ch. 12][1]

$$\hat{h}_{\text{eq}}(s) = a y_c + b, \tag{5}$$

where

$$a = \frac{\text{Cov}\left(y_c, h_{\text{eq}}(s)\right)}{\text{Var}\left(y_c\right)} \text{ and } b = \left(1 - \sqrt{P_c} a\right) \mathbb{E}\left[h_{\text{eq}}(s)\right], \tag{6}$$

$\text{Cov}(.,.)$ denotes covariance, and $\text{Var}(.)$ denotes variance. To derive a and b, we need the statistics of $h_{\text{eq}}(s)$, which are different

[1]The two observations y_c and y_s can be used to obtain a more refined channel estimate of $h_{\text{eq}}(s)$. However, the gains turn out to be marginal.

from that of $h_{\text{eq}}(1), h_{\text{eq}}(2), \ldots, h_{\text{eq}}(K)$ due to selection. Since $h_1 \ldots, h_M$ are i.i.d., we can prove that $h_{\text{eq}}(1), h_{\text{eq}}(2), \ldots, h_{\text{eq}}(K)$ are i.i.d. complex normal RVs with mean 0 and variance $M\beta_g\beta_h$. Let $\sigma_y^2 = P_{\text{s}}M\beta_g\beta_h + \sigma^2$ denote the variance of y_i. The PDF of $h_{\text{eq}}(s)$ is as follows.

Theorem 1. *The PDF of $h_{eq}(s) = e_{rs} + je_{is}$ is given by*

$$f_{e_{rs},e_{is}}(x,z) = \frac{K\sigma_y^2}{\pi M\beta_g\beta_h}\sum_{l=0}^{K-1}\binom{K-1}{l}\frac{(-1)^l}{\sigma_y^2 + l\sigma^2} \times \exp\left(\frac{-(l+1)\sigma_y^2}{\left(\sigma_y^2 + l\sigma^2\right)M\beta_g\beta_h}\left(x^2+z^2\right)\right),\ x,z\in\mathbb{R}. \quad (7)$$

Proof. The proof is given in Appendix 6.1. □

The symmetric form of the PDF in Theorem 1 implies that $\mathbb{E}\left[h_{\text{eq}}(s)\right] = 0$. Hence, $b = 0$. Thus, from (5) and (6), the selection-aware estimator is given by

$$\hat{h}_{\text{eq}}(s) = \frac{\sqrt{P_{\text{c}}}\mathbb{E}\left[|h_{\text{eq}}(s)|^2\right]}{P_{\text{c}}\mathbb{E}\left[|h_{\text{eq}}(s)|^2\right] + \sigma^2}y_{\text{c}}. \quad (8)$$

A selection-unaware approach would have instead led to the following estimator: $\hat{h}_{\text{eq}}(s) = \frac{\sqrt{P_{\text{c}}}M\beta_g\beta_h}{P_{\text{c}}M\beta_g\beta_h + \sigma^2}y_{\text{c}}$.

Theorem 1 leads to the following novel closed-form expression for the passive beamforming gain $B_{\text{g}} = \mathbb{E}\left[|h_{\text{eq}}(s)|^2\right]$.

Corollary 1. *The passive beamforming gain B_g is given by*

$$B_g = \frac{M\beta_g\beta_h\left(P_sM\beta_g\beta_h\left[\psi(K+1)+\gamma\right] + \sigma^2\right)}{P_sM\beta_g\beta_h + \sigma^2}, \quad (9)$$

where $\psi(.)$ is the digamma function [21, Ch. 6] and γ is the Euler-Mascheroni constant [21, Ch. 6].

From (9), we can show that B_{g} is a concave function of P_{s}. It is also a concave function of K. Thus, there is a diminishing increase in the beamforming gain when P_{s} or K increase.

3.2. Achievable Rate with Imperfect Channel Estimates

The achievable rate R in the presence of channel estimation errors, after accounting for the training, control signaling, and feedback overheads, is given by [22][2]

$$R = \frac{d}{T_{\text{c}}}\mathbb{E}\left[\log_2\left(1 + \frac{P_{\text{d}}\left|\hat{h}_{\text{eq}}(s)\right|^2}{\sigma^2 + P_{\text{d}}\sigma^2_{\hat{h}_{\text{eq}}(s)}}\right)\right], \quad (10)$$

where $\sigma^2_{\hat{h}_{\text{eq}}(s)} = \mathbb{E}\left[\left|h_{\text{eq}}(s) - \hat{h}_{\text{eq}}(s)\right|^2\right]$ is the mean square error (MSE) of the estimate of $h_{\text{eq}}(s)$, d is the number of data symbols, and $T_{\text{c}} = d + K + 1 + 2\tau_{\text{f}} + \tau_{\text{c}}$ is the coherence interval duration in symbols. Using the Jensen's inequality, we get the following bound:

$$R \le \frac{d}{T_{\text{c}}}\log_2\left(1 + \frac{P_{\text{d}}\mathbb{E}\left[\left|\hat{h}_{\text{eq}}(s)\right|^2\right]}{\sigma^2 + P_{\text{d}}\sigma^2_{\hat{h}_{\text{eq}}(s)}}\right). \quad (11)$$

[2]The achievable rate expression in (10) is an approximation because $h_{\text{eq}}(s)$ is not a complex normal RV. However, it is accurate for smaller K and at lower SNRs.

We shall see in Section 4 that the above bound is tight for large M due to channel hardening. From (4) and (8), we can show that

$$\mathbb{E}\left[\left|\hat{h}_{\text{eq}}(s)\right|^2\right] = \frac{P_{\text{c}}B_{\text{g}}^2}{P_{\text{c}}B_{\text{g}} + \sigma^2},\ \sigma^2_{\hat{h}_{\text{eq}}(s)} = \frac{B_{\text{g}}\sigma^2}{P_{\text{c}}B_{\text{g}} + \sigma^2}. \quad (12)$$

Substituting these in (11) yields the following result.

Theorem 2. *The achievable rate of the codebook-based IRS-aided system in the presence of channel estimation errors is bounded by*

$$R \le \frac{d}{T_c}\log_2\left(1 + \frac{P_dP_cB_g^2}{\sigma^2\left[\left(P_d + P_c\right)B_g + \sigma^2\right]}\right), \quad (13)$$

where B_g is given in (9). □

The total power P_{T} consumed for the pilot and data transmissions is equal to $KP_{\text{s}} + P_{\text{c}} + dP_{\text{d}}$. Based on (13), the problem of maximizing the upper bound on R subject to the total power constraint at AP can be stated as

$$\max_{P_{\text{s}},P_{\text{c}},P_{\text{d}}}\left\{\frac{P_{\text{d}}P_{\text{c}}B_{\text{g}}^2}{\sigma^2\left[\left(P_{\text{d}} + P_{\text{c}}\right)B_{\text{g}} + \sigma^2\right]}\right\}, \quad (14)$$

$$\text{s.t. } KP_{\text{s}} + P_{\text{c}} + dP_{\text{d}} = P_{\text{T}}, \quad (15)$$

$$P_{\text{s}} \ge 0, P_{\text{c}} \ge 0, P_{\text{d}} \ge 0. \quad (16)$$

Note that the selection pilot power P_{s} appears implicitly through B_{g}. We solve the above non-convex problem numerically.

4. NUMERICAL RESULTS AND DISCUSSIONS

We set $M = 128$ and consider $T_{\text{c}} = 400$ symbols. We consider the simplified path loss model [6] with $\beta_g = -34$ dB and $\beta_h = -54$ dB. The noise variance is $\sigma^2 = -113.8$ dB. We set $\tau_{\text{f}} = \tau_{\text{c}} = 1$. The results are averaged over 10000 fade realizations. We consider a DFT-based codebook at the IRS.

We benchmark the proposed optimal power allocation-based codebook scheme with the following:

i) Equal Power Allocation (EPA): The total power is equally allocated to the pilots and data. Furthermore, the unordered statistics of the beamforming gain are used to estimate $h_{\text{eq}}(s)$. The training, signaling, and feedback overhead is $K + 1 + 2\tau_f + \tau_c$.

ii) Estimation-based Scheme [3]: We use the DFT-based estimation scheme since it outperforms the on-off scheme [3]. Its training, signaling, and feedback overhead is equal to $M + M\tau_c + 1 + \tau_f$.

iii) Genie-aided Scheme: The selection of reflection pattern is taken to be noise-free and $h_{\text{eq}}(s)$ is known perfectly to the AP and the user. There is no training, signaling, and feedback overhead.

iv) Random Phase Shift (RPS): A random phase shift is configured at each IRS element. Thus, no selection pilots are needed. The training, signaling, and feedback overhead is $1 + \tau_f$.

Figure 2 plots the achievable rate of all the schemes as a function of the average SNR $P_TM\beta_g\beta_h/(T_{\text{c}}\sigma^2)$. The proposed scheme achieves a higher rate than EPA and RPS for all SNRs. The gap between it and the genie-aided scheme is small and decreases as the SNR increases. Only at low SNRs, does the estimation-based scheme marginally outperform the proposed scheme. This is because the beamforming gain of the former is proportional to M^2 [4], while that of the latter is proportional to $M\psi(K+1)$ (see (9)). At high SNRs, the large training and control overheads cause the estimation-based scheme's rate to be lower than even RPS.

Figure 3(a) plots the ratio of the optimal selection pilots' power to the total power as a function of the average SNR for different values of K. It compares them with EPA, where this ratio is equal to

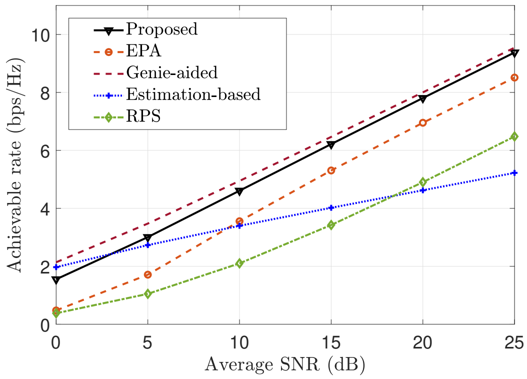
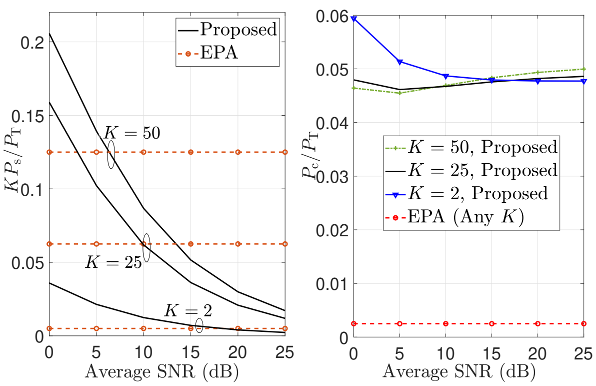


Fig. 2. Performance Benchmarking: Achievable rate as a function of the average SNR ($K = 25$).



(a) Selection pilots power (b) Demodulation pilot power

Fig. 3. Pilot power allocation as a function of the average SNR.

K/T_c. For the proposed scheme, we see that the ratio decreases for all K as the average SNR increases. At low SNRs or larger K, it allocates relatively more power to the selection pilots to improve the odds that the optimal reflection pattern is selected. At high SNRs, it sets aside more power for the demodulation pilot and the data. Figure 3(b) plots the corresponding ratio of the optimal demodulation pilot power to the total power. Notably, the proposed scheme allocates 13 to 14 dB more power to the demodulation pilot than EPA at all SNRs. Also, we observe that $P_c \gg P_s$. For example, at $K = 50$ and SNR of 0 dB, $P_c/P_s = 15$.

Figure 4 plots the achievable rate and the upper bound in (13) as a function of K for different T_c. We observe that the upper bound is tight, which is due to channel hardening. For small K, the achievable rate increases as K increases because the beamforming gain increases. However, for larger K, the rate decreases because of the increase in the training overhead. Only for large coherence interval durations does the estimation-based scheme have a higher rate.

5. CONCLUSIONS

For the orthogonal codebook-based IRS system, we proposed a novel training scheme that could allocate more power to the demodulation pilot than a selection pilot. We derived a tight upper bound for its achievable rate and a closed-form expression for the average beamforming gain. The analysis accounted for imperfect channel estimates, which affected both selection and coherent demodulation. The proposed scheme achieved a higher rate than the estimation-based scheme except at low SNRs or when the coherence interval duration was much greater than the number of IRS elements. Future research directions include considering multiple antennas, and considering correlation between the cascaded channels. With the direct link, an interesting problem is determining the pilot powers as a function of the link's strength relative to the cascaded links.

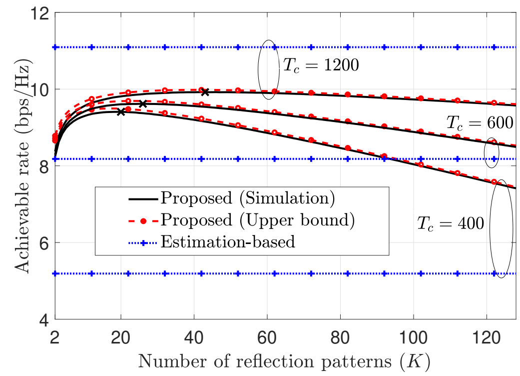


Fig. 4. Achievable rate as a function of the number of reflection patterns (K). The maximum rate is shown by the marker 'x'.

6. APPENDIX

6.1. Outline of Proof of Theorem 1

Due to space constraints, we only show the key steps. Let $h_{\text{eq}}(m) = e_{rm} + je_{im}$, for $m \in \{1, 2, \ldots, K\}$. The cumulative distribution function (CDF) of $h_{\text{eq}}(s)$ can be written as $F_{e_{rs},e_{is}}(x, z) = \Pr(e_{rs} \le x, e_{is} \le z)$. It is given by

$$F_{e_{rs},e_{is}}(x, z) = \sum_{i=1}^{K} \Pr(e_{rs} \le x, e_{is} \le z, s = i), \quad (17)$$

$$= K\Pr(e_{r1} \le x, e_{i1} \le z, s = 1). \quad (18)$$

Let $\mathcal{K}_1 = \{2, 3, \ldots, K\}$. Using (3), conditioning on e_{r1}, e_{i1} and taking expectation over them, the probability in (18) can be written as

$$\mathbb{E}\left[\mathbb{1}_{\{e_{r1} \le x, e_{i1} \le z,\}} \Pr\left(|y_1|^2 > |y_i|^2, \forall i \in \mathcal{K}_1 \middle| e_{r1}, e_{i1}\right)\right], \quad (19)$$

where $\mathbb{1}_{\{.\}}$ denotes the indicator function. Conditioned on e_{r1}, e_{i1}, we can show that $|y_1|^2$ is a non-central chi-square RV with PDF $e^{-(w+P_s e^2)/\sigma^2} I_0\left(\sqrt{4P_s e^2 w/\sigma^4}\right)/\sigma^2$, for $w \ge 0$, where $e = \sqrt{e_{r1}^2 + e_{i1}^2}$ and $I_0(.)$ is the zeroth-order modified Bessel function of the first kind [23, Ch. 5]. Using this, we can then show that

$$\Pr\left(|y_1|^2 > |y_i|^2, \forall i \in \mathcal{K}_1 | e_{r1}, e_{i1}\right)$$
$$= \frac{1}{\sigma^2} \int_0^\infty e^{-\frac{w+P_s e^2}{\sigma^2}} I_0\left(\sqrt{\frac{4P_s e^2 w}{\sigma^4}}\right)\left(1 - e^{-\frac{w}{\sigma_y^2}}\right)^{K-1} dw. \quad (20)$$

Expanding the term $\left(1 - \exp\left(-w/\sigma_y^2\right)\right)^{K-1}$ and using the identities in [24, (2) and (9)] and [25, (2.33)] simplifies (20). Substituting it in (19) and averaging over e_{r1} and e_{i1} yields the expression for $F_{e_{rs},e_{is}}(x, z)$. Differentiating this with respect to x and z gives us the expression for PDF in (7).

7. REFERENCES

[1] Ertugrul Basar, Marco Di Renzo, Julien De Rosny, Merouane Debbah, Mohamed-Slim Alouini, and Rui Zhang, "Wireless communications through reconfigurable intelligent surfaces," *IEEE Access*, vol. 7, pp. 116753–116773, Aug. 2019.

[2] Marco Di Renzo, Alessio Zappone, Merouane Debbah, Mohamed-Slim Alouini, Chau Yuen, Julien de Rosny, and Sergei Tretyakov, "Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead," *IEEE Journal on Selected Areas in Communications*, vol. 38, no. 11, pp. 2450–2525, Nov. 2020.

[3] Beixiong Zheng and Rui Zhang, "Intelligent reflecting surface-enhanced OFDM: Channel estimation and reflection optimization," *IEEE Wireless Communications Letters*, vol. 9, no. 4, pp. 518–522, Apr. 2020.

[4] Qingqing Wu and Rui Zhang, "Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming," *IEEE Transactions on Wireless Communications*, vol. 18, no. 11, pp. 5394–5409, Nov. 2019.

[5] Yifei Yang, Beixiong Zheng, Shuowen Zhang, and Rui Zhang, "Intelligent reflecting surface meets OFDM: Protocol design and rate maximization," *IEEE Transactions on Communications*, vol. 68, no. 7, pp. 4522–4535, Jul. 2020.

[6] Jiancheng An, Li Wang, Chao Xu, Lu Gan, and Lajos Hanzo, "Optimal pilot power based channel estimation improves the throughput of intelligent reflective surface assisted systems," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 12, pp. 16202–16206, Dec. 2020.

[7] Bharath Shamasundar, Negar Daryanavardan, and Aria Nosratinia, "Channel training & estimation for reconfigurable intelligent surfaces: Exposition of principles, approaches, and open problems," *IEEE Access*, vol. 11, pp. 6717–6734, Jan. 2023.

[8] Abdelrahman Taha, Muhammad Alrabeiah, and Ahmed Alkhateeb, "Deep learning for large intelligent surfaces in millimeter wave and massive MIMO systems," in *Proc. IEEE Global Communications Conference (GLOBECOM)*, Dec. 2019, pp. 1–6.

[9] Özgecan Özdoğan and E. Björnson, "Deep learning-based phase reconfiguration for intelligent reflecting surfaces," in *Proc. Asilomar Conference on Signals, Systems, and Computers*, Nov. 2020, pp. 707–711.

[10] Yu Jin, Jiayi Zhang, Chongwen Huang, Liang Yang, Huahua Xiao, Bo Ai, and Zhiqin Wang, "Multiple residual dense networks for reconfigurable intelligent surfaces cascaded channel estimation," *IEEE Transactions on Vehicular Technology*, vol. 71, no. 2, pp. 2134–2139, Feb. 2022.

[11] Linglong Dai and Xiuhong Wei, "Distributed machine learning based downlink channel estimation for ris assisted wireless communications," *IEEE Transactions on Communications*, vol. 70, no. 7, pp. 4900–4909, Jul. 2022.

[12] Jiancheng An, Chao Xu, Li Wang, Yusha Liu, Lu Gan, and Lajos Hanzo, "Joint training of the superimposed direct and reflected links in reconfigurable intelligent surface assisted multiuser communications," *IEEE Transactions on Green Communications and Networking*, vol. 6, no. 2, pp. 739–754, Jun. 2022.

[13] Jiancheng An, Chao Xu, Qingqing Wu, Derrick Wing Kwan Ng, Marco Di Renzo, Chau Yuen, and Lajos Hanzo, "Codebook-based solutions for reconfigurable intelligent surfaces and their open challenges," *To appear in IEEE Wireless Communications*, 2023.

[14] Yutong Zhang, Boya Di, Hongliang Zhang, Miaomiao Dong, Lu Yang, and Lingyang Song, "Dual codebook design for intelligent omni-surface aided communications," *IEEE Transactions on Wireless Communications*, vol. 21, no. 11, pp. 9232–9245, Nov. 2022.

[15] Jinghe Wang, Wankai Tang, Shi Jin, Chao-Kai Wen, Xiao Li, and Xiaolin Hou, "Hierarchical codebook-based beam training for ris-assisted mmwave communication systems," *IEEE Transactions on Communications*, vol. 71, no. 6, pp. 3650–3662, Jun. 2023.

[16] Xiuhong Wei, Linglong Dai, Yajun Zhao, Guanghui Yu, and Xiangyang Duan, "Codebook design and beam training for extremely large-scale ris: Far-field or near-field?," *China Communications*, vol. 19, no. 6, pp. 193–204, Jun. 2022.

[17] Peilan Wang, Jun Fang, Wei Zhang, and Hongbin Li, "Fast beam training and alignment for IRS-assisted millimeter wave/terahertz systems," *IEEE Transactions on Wireless Communications*, vol. 21, no. 4, pp. 2710–2724, Apr. 2022.

[18] Vahid Jamali, Marzieh Najafi, Robert Schober, and H. Vincent Poor, "Power efficiency, overhead, and complexity tradeoff of IRS codebook design—quadratic phase-shift profile," *IEEE Commun. Lett.*, vol. 25, no. 6, pp. 2048–2052, Jun. 2021.

[19] Xing Jia, Jiancheng An, Hao Liu, Hongshu Liao, Lu Gan, and Chau Yuen, "Environment-aware codebook for reconfigurable intelligent surface-aided miso communications," *IEEE Wireless Communications Letters*, vol. 12, no. 7, pp. 1174–1178, Jul. 2023.

[20] Steven M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice-Hall, Inc., USA, first edition, 1993.

[21] Irene A. Stegun Milton Abramowitz, *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*, Dover Publications, first edition, 1965.

[22] B. Hassibi and B.M. Hochwald, "How much training is needed in multiple-antenna wireless links?," *IEEE Transactions on Information Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.

[23] Andreas F. Molisch, *Wireless Communications*, Wiley Publishing, second edition, 2011.

[24] A. H. Nuttall, *Some Integrals Involving the Q-Function*, Defense Technical Information Center, first edition, 1972.

[25] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Elsevier/Academic Press, Amsterdam, seventh edition, 2007.