Distributed Energy-Efficient Power Allocation for a Spectrum Sharing Cell-Free System

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***Abstract*—We focus on a spectrum sharing based secondary cell-free system, which enables efficient utilization of the limited spectrum. For it, we develop a distributed downlink power allocation algorithm, in which each access point (AP) computes only its power coefficients locally, to improve energy efficiency (EE) while satisfying the interference constraints imposed by the primary system. Each AP solves smaller number of variables in parallel compared to centralized approach in which all power coefficients are computed centrally. Hence, it is computationally efficient and is scalable in terms of the number of APs. For the simulation setup considered with correlated Rayleigh fading, the proposed distributed approach achieves up-to 85% of the centralized EE. Furthermore, it achieves better performance compared to other distributed approaches.**

***Index Terms*—Cell-free, spectrum sharing, energy efficiency, distributed, power allocation.**

I. Introduction

Cell-free massive multiple input multiple output (MIMO) systems in which multiple geographically distributed access points (APs) cooperate to serve a large number of users can provide uniform service to the users [1]. In the centralized approach, the APs send received signals to the central processing unit (CPU), which performs channel estimation, precoding, and power allocation. It requires a large amount of signaling between the APs and CPU and increases complexity at the CPU. In the distributed approach, which is scalable, the APs perform some of the tasks locally and reduce the signaling overhead and complexity at the CPU.

Spectrum sharing, which is widely accepted by the spectrum regulators, is crucial to enable new wireless technologies and efficient use of the scarce spectrum. Federal Communications Commission allows spectrum sharing in sub-6 GHz bands, especially for low-power indoor operations [2]. In the underlay mode of spectrum sharing, the secondary system transmits concurrently with the primary system, while satisfying the interference constraint imposed by the primary system [3], [4].

The energy efficiency (EE), which is equal to the number of bits that can be transmitted using one Joule of energy, is an important performance metric of a communication system. Transmit power allocation algorithms, which play a crucial role in achieving better EE, are developed for a cell-free system in [5], [6]. A max-min fairness based power allocation is studied for an underlay cell-free system is studied in [4]. The sum-rate maximization is studied in [7], [8]. The power allocation of a centralized cell-free secondary system to maximize its EE is developed in [9]. Its complexity increases as the number of secondary users or the number of APs increases. However, the distributed power control that maximize EE is not developed for a secondary cell-free system.

Focus and Contributions: We consider a downlink secondary cell-free massive MIMO system that operates in distributed approach and shares spectrum with a primary massive MIMO system. The L secondary APs perform minimum mean square error (MMSE) channel estimation, maximal ratio (MR) precoding, and allocates power to K_s secondary users locally to maximize EE. We focus on reducing the computational complexity and scalability issues involved in the centralized power allocation. We consider practically relevant correlated Rayleigh fading and average interference constraint to protect the primary system from excessive interference.

i) Power allocation problem to maximize EE involves LK_s power allocation variables and needs to be solved at the CPU. We modify the objective and constraints of the centralized optimization problem to formulate a distributed problem with K_s power allocation variables of one AP. It can be solved locally at each AP. First, we propose a convex lower bound based iterative algorithm whose complexity increase only with K_s instead of LK_s as in centralized approach.

ii) We then propose a complete power allocation algorithm that exploits power scaling to further improve the EE and use distributed equal power to reduce the complexity. This distributed approach obtains a trade-off between the EE performance and the computational complexity. It is scalable in terms of number of APs.

iii) For the simulation setup considered, we show that our distributed approach reaches 85% EE of the centralized approach while allowing parallel computations. As distributed EE power allocation is not studied in the literature for our system model, we benchmark the performance with the adaptations of the existing simpler power allocations schemes. We shows that the proposed approach yields a better performance.

II. System Model and Problem Formulation

Our system model, in which a secondary cell-free system transmits concurrently with a primary massive MIMO system, is shown in Fig. 1. The M antenna primary base station (BS) serves K_p single antenna primary users. The secondary cell-free system, which shares spectrum with the primary, has L distributed APs with N antennas each. Each AP performs power allocation locally to serve all the K_s single antenna

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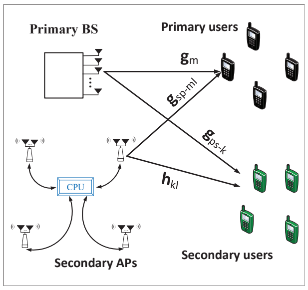


Fig. 1. Secondary cell-free system that transmits concurrently with a primary massive MIMO system.

secondary users. Furthermore, there is no communication among the APs to perform power allocation. Both primary and secondary systems operate in time division duplex mode.

Let $\mathbf{h}_{kl} \in \mathbb{C}^{N\times 1}$ and $\mathbf{g}_{\text{sp}-ml} \in \mathbb{C}^{N\times 1}$ denote the complex channel gain vectors from the l^{th} secondary AP to the k^{th} secondary user and to the m^{th} primary user. We consider the correlated Rayleigh fading channel model. Let $\mathbf{R}_{kl}$ and $\mathbf{B}_{\text{sp}-ml}$ denote the covariance matrices of $\mathbf{h}_{kl}$ and $\mathbf{g}_{\text{sp}-ml}$, respectively. Therefore, $\mathbf{h}_{kl} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_{kl}\right)$ and $\mathbf{g}_{\text{sp}-ml} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{B}_{\text{sp}-ml}\right)$. Let $\mathbf{g}_{\text{ps}-k} \in \mathbb{C}^{M\times 1}$ and $\mathbf{g}_{m} \in \mathbb{C}^{M\times 1}$ denote the complex channel gain vectors from the primary BS to the k^{th} secondary user and to the m^{th} primary user, respectively. Let $\mathbf{B}_{\text{ps}-k}$ and $\mathbf{D}_{m}$ denote the covariance matrices of $\mathbf{g}_{\text{ps}-k}$ and $\mathbf{g}_{m}$, respectively. Therefore, $\mathbf{g}_{\text{ps}-k} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{B}_{\text{ps}-k}\right)$ and $\mathbf{g}_{m} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{D}_{m}\right)$. We assume that the channel gains of different links are independent of each other. The secondary APs know the estimates of the channel gains to the secondary users and statistics of the channel gains to the primary users. CPU only knows the channel statistics [4], [10].

Channel Estimation: APs perform MMSE channel estimation described in [1, Sec. 4.2]. The number of symbols in the coherence interval and pilot sequence are denoted by τ_{c} and τ_{p}, respectively. Let $\phi_i \in \mathbb{C}^{\tau_{\text{p}}\times 1}$, for $i = 1, \ldots, \tau_{\text{p}}$ denote unit norm orthogonal pilot sequences shared among the primary and secondary systems. We consider pilot contamination. The secondary user k transmits the pilot sequence ϕ_{t_k}. The same pilot sequence is transmitted by sets of primary and secondary users denoted by $\mathcal{M}_k$ and $\mathcal{N}_k$, respectively. Let $a \geq 0$ and $b \geq 0$ denote the pilot powers used by the secondary and primary users, respectively. We project the pilot signal received at the l^{th} secondary AP onto ϕ_{t_k}. It is given by

$$\mathbf{y}^{\text{p}}_{t_k l} = \sqrt{a} \sum_{i \in \mathcal{N}_k} \mathbf{h}_{il} + \sqrt{b} \sum_{j \in \mathcal{M}_k} \mathbf{g}_{\text{sp}-jl} + \mathbf{z}_{t_k l}, \tag{1}$$

where $\mathbf{z}_{t_k l}$ is the projected noise vector distributed as $\mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_N\right)$. Now, using $\mathbf{y}^{\text{p}}_{t_k l}$, the MMSE estimate

$$\widehat{\mathbf{h}}_{kl} = \sqrt{a} \mathbf{R}_{kl} A^{-1}_{t_k l} \mathbf{y}^{\text{p}}_{t_k l}, \tag{2}$$

where $\mathbf{A}_{t_k l} = a \sum_{i \in \mathcal{N}_k} \mathbf{R}_{il} + b \sum_{j \in \mathcal{M}_k} \mathbf{B}_{\text{sp}-jl} + \sigma^2 \mathbf{I}_N$ [1]. From (2), we see that $\widehat{\text{h}}_{kl} \sim \mathcal{CN}\left(\mathbf{0}, \widehat{\mathbf{R}}_{kl}\right)$, where $\widehat{\mathbf{R}}_{kl} = a \mathbf{R}_{kl} A^{-1}_{t_k l} \mathbf{R}_{kl}$ and the estimation error $\widetilde{\mathbf{h}}_{kl} = \mathbf{h}_{kl} - \widehat{\mathbf{h}}_{kl} \sim \mathcal{CN}\left(\mathbf{0}, \widetilde{\mathbf{R}}_{kl}\right)$, where $\widetilde{\mathbf{R}}_{kl} = \mathbf{R}_{kl} - \widehat{\mathbf{R}}_{kl}$.

Signal-to-Interference-Plus-Noise-Ratio (SINR) Computation: Let $p_{il} \geq 0$ and $\mathbf{a}_{il}$ denote the power allocated and the precoding vector used to transmit unit power symbol q_i from l^{th} secondary AP to i^{th} secondary user. Let $\mathbf{p} = \left[p_{11}, \ldots, p_{Ks1}, \ldots, p_{1L}, \ldots, p_{KsL}\right]^T$ denote the power allocation vector. Channel estimation based MR precoding is done at the secondary APs. Hence, $\mathbf{a}_{il} = \widehat{\mathbf{h}}_{il} \Big/ \sqrt{\text{Trace}\left(\widehat{\mathbf{R}}_{il}\right)}$.

The signal y_k received at the secondary user k when the AP l transmits $\mathbf{x}_l = \sum_{i=1}^{K_s} \sqrt{p_{il}} \mathbf{a}_{il} q_i, l = 1, \ldots, L$, is given by

$$y_k = \sum_{l=1}^{L} \sqrt{p_{kl}} \mathbf{h}^H_{kl} \mathbf{a}_{kl} q_k + \sum_{\substack{i=1 \\ i \neq k}}^{K_{\text{s}}} \sum_{l=1}^{L} \sqrt{p_{il}} \mathbf{h}^H_{kl} \mathbf{a}_{il} q_i + d_k + z_k,$$

where $z_k \sim \mathcal{CN}\left(0, \sigma^2\right)$ is additive white Gaussian noise and d_k denotes the interference from the primary massive MIMO BS with collocated antennas. We assume d_k to be negligibly small due to independence of channel gains and favorable propagation. The secondary user k only knows $\mathbb{E}\left\{\mathbf{h}^H_{kl} \mathbf{a}_{kl}\right\}$, for $l = 1, \ldots, L$, which are the channel statistics. With this statistical channel knowledge, the effective SINR $\Gamma_k\left(\mathbf{p}\right)$ for the use and forget bound can be derived using techniques in [1, Ch. 6]. Upon simplification, we get

$$\Gamma_k\left(\mathbf{p}\right) = \frac{\left(\sum_{l=1}^{L} \sqrt{p_{kl} \,\text{Trace}\left(\widehat{\mathbf{R}}_{kl}\right)}\right)^2}{\sum\limits_{i=1}^{K_s} \sum\limits_{l=1}^{L} p_{kl} n_{ikl} + \sum\limits_{i=1, i\neq k}^{K_s} \left(\sum\limits_{l=1}^{L} \sqrt{p_{il}} m_{ikl}\right)^2 + \sigma^2_k}, \tag{3}$$

where σ^2_k is the interference-plus-noise-power,

$$n_{ikl} = \text{Trace}\left(\widehat{\mathbf{R}}_{il} \mathbf{R}_{kl}\right) \Big/ \text{Trace}\left(\widehat{\mathbf{R}}_{il}\right), \tag{4}$$

$$m_{ikl} = \mathbb{I}_{\mathcal{N}_k}(i) \text{Trace}\left(\mathbf{R}_{il} \boldsymbol{A}^{-1}_{t_i l} \mathbf{R}_{kl}\right) \Big/ \sqrt{\text{Trace}\left(\widehat{\mathbf{R}}_{il}\right)}, \tag{5}$$

with $\mathbb{I}_{\mathcal{N}_k}(i) = 1$, when the secondary users i and k use the same pilot sequence. Note that $m^2_{kkl} = \text{Trace}\left(\widehat{\mathbf{R}}_{kl}\right)$.

A. Objective and Constraints

The net achievable rate of the secondary user k is given by

$$\text{SE}_k(\mathbf{p}) = \left(1 - \tau_p/\tau_c\right) \log_2\left(1 + \Gamma_k\left(\mathbf{p}\right)\right) \text{ bit/sec/Hz}. \tag{6}$$

Let η and Cp denote the inverse power amplifier efficiency and the total circuit power consumption, respectively. Then, the total power consumed at all the secondary APs is given by

$$\overline{P}_T(\mathbf{p}) = \eta \sum_{i=1}^{K_s} \sum_{l=1}^{L} p_{il} + \text{C}_{\text{p}}. \tag{7}$$

The EE, which is the ratio of sum rate and total power consumed, as a function of $\mathbf{p}$, is given by

$$\text{EE}(\mathbf{p}) = B \sum_{k=1}^{K_s} \text{SE}_k(\mathbf{p}) \Big/ \overline{P}_T(\mathbf{p}), \tag{8}$$

where B is the system bandwidth. The EE maximization is subject to the following two constraints

i) The peak transmit power constraint at each AP, motivated by the power amplifier limitations, limits the total average transmit power to be below $P_{\max}$. It is given by

$$\sum_{i=1}^{K_s} p_{il} \leq P_{\max},\ l = 1, \cdots, L. \tag{9}$$

ii) The average interference constraint, which protects the primary users from the excessive interference, limits the total average interference seen at each primary user to be less than or equal to I_{th} [4], [11]. In the distributed approach, each secondary AP only knows its power allocation coefficients. Hence, it can only compute the interference caused due to its transmissions and not the total interference. To overcome this issue, we propose distributed interference constraints at each AP. We limit the interference due to transmissions of each AP to be below I_{th}/L. This ensures that the total interference seen at each primary user due to L secondary APs is below I_{th}.

The interference signal received at m^{th} primary user due to transmission from l^{th} AP is given by $y_{ml}^{\text{sp}} = \mathbf{g}_{\text{sp}-ml}^H \mathbf{x}_l$. The power of this interference signal is given by $\mathbb{E}\left\{|y_{ml}^{\text{sp}}|^2\right\} = \sum_{i=1}^{K_s} p_{il} b_{iml}$, where $b_{iml} = \text{Trace}\left(\widehat{\mathbf{R}}_{il}\mathbf{B}_{\text{sp}-ml}\right) / \text{Trace}\left(\widehat{\mathbf{R}}_{il}\right)$. Therefore, the set of average interference constraints imposed at l^{th} secondary AP are given by

$$\sum\nolimits_{i=1}^{K_s} p_{il} b_{iml} \leq I_{\text{th}}/L, \quad m = 1, \ldots, K_p. \tag{10}$$

III. Distributed Power Allocation Algorithm

The EE in (8) is a function of the power allocation coefficients of all the APs. Hence, their joint optimization to maximize EE has to be done at the CPU. We will now modify this objective function to develop a distributed power allocation algorithm that can be implemented at each AP.

Each AP formulates a distributed objective function, which depends only on its power allocation coefficients, assuming that all the other APs are transmitting with equal power that satisfies the peak power constraint and total average interference constraint, i.e., $\sum_{l=1}^{L}\sum_{i=1}^{K_s} p_{il} b_{iml} \leq I_{\text{th}}$ for $m = 1, \ldots, K_{\text{p}}$. It is given by

$$P_{\text{eq}} = \min\left\{P_{\max}/K_{\text{s}}, I_{\text{th}}/V_1, \ldots, I_{\text{th}}/V_{K_{\text{p}}}\right\}, \tag{11}$$

where $V_m = \sum_{l=1}^{L}\sum_{i=1}^{K_{\text{s}}} b_{iml}$. The CPU computes P_{eq} and sends to the APs. Let $\mathbf{p}_r = [p_{1r}, p_{2r}, \ldots, p_{K_s r}]^T$ denote the vector of power allocation coefficients at r^{th} AP. At AP r, we rewrite the total power consumed in terms of p_{ir} and p_{il}, for $l \neq r$. Substituting $p_{il} = P_{\text{eq}}$, for $l \neq r$ in (7), we obtain $\overline{P}_T^d(\mathbf{p}_r)$, which is function of only $\mathbf{p}_r$:

$$\overline{P}_T^d(\mathbf{p}_r) = \eta \sum\nolimits_{i=1}^{K_s} p_{ir} + \eta(L-1)K_s P_{eq} + \text{C}_{\text{p}}. \tag{12}$$

Now to obtain a distributed formulation of $\text{SE}_k(\mathbf{p})$ in the numerator of the EE in (8), we focus on centralized effective SINR $\Gamma_k(\mathbf{p})$ of the k^{th} secondary user in (3). We split the numerator of $\Gamma_k(\mathbf{p})$ in terms of p_{ir} and p_{il}, for $l \neq r$ as follows: $\left(\sqrt{p_{kr}} m_{kkr} + \sum_{l=1, l\neq r}^{L} \sqrt{p_{kl}} m_{kkl}\right)^2$. Upon substituting $p_{il} = P_{\text{eq}}$, for $l \neq r$, it reduces to $\left(\sqrt{p_{kr}} m_{kkr} + \sqrt{P_{\text{eq}}} \sum_{l=1, l\neq r}^{L} m_{kkl}\right)^2$. Similarly, the first term in the denominator, i.e., $\sum_{i=1}^{K_s}\sum_{l=1}^{L} p_{kl} n_{ikl}$, reduces to $\sum_{i=1}^{K_s} p_{kr} n_{ikr} + P_{\text{eq}} \sum_{l=1, l\neq r}^{L}\sum_{i=1}^{K_s} n_{ikl}$. Performing a similar simplification to the second term in the denominator of $\Gamma_k(\mathbf{p})$, which arises due to the pilot contamination, we get

$$\sum\nolimits_{i=1, i\neq k}^{K_s}\left(\sqrt{p_{ir}} m_{ikr} + \sqrt{P_{eq}} \sum\nolimits_{l=1, l\neq r}^{L} m_{ikl}\right)^2. \tag{13}$$

Substituting the above simplifications in (3), yields the modified SINR, which we denote as $\Gamma_k^d(\mathbf{p}_r)$. It is given in (14).

Substituting the modified SINR in (14) and the modified power in (12) in the objective function in (8) yields

$$\frac{B\left(1 - \frac{\tau_p}{\tau_c}\right)\sum_{k=1}^{K_s} \log_2\left(1 + \Gamma_k^d(\mathbf{p}_r)\right)}{\eta \sum_{i=1}^{K_s} p_{ir} + \eta(L-1)K_s P_{eq} + \text{C}_{\text{p}}}, \tag{15}$$

which is a function of only the power allocation coefficients corresponding to AP r. Therefore, our distributed optimization problem at AP r that maximizes the above modified objective function subject to the constraints is given by

$$\begin{aligned} \mathcal{P}_d: \quad & \max_{\mathbf{p}_r} \quad (15), \\ & \text{s.t.} \quad (9),\ (10),\ p_{ir} \geq 0, i = 1, \ldots, K_s. \end{aligned} \tag{16}$$

Although (9) and (10) are convex, the objective function (15) is non-convex. Hence, $\mathcal{P}_d$ is non-convex.

A. Distributed Optimal Power Allocation

To solve the above non-convex optimization problem $\mathcal{P}_d$ at AP r, we obtain a convex lower bound on (15) and maximize it. To do that, we focus on its numerator and rewrite

$$\sum_{k=1}^{K_s} \log_2\left(1 + \Gamma_k^d(\mathbf{p}_r)\right) = f_1(\mathbf{p}_r) - f_2(\mathbf{p}_r), \tag{17}$$

where $f_1(\mathbf{p}_r) = \sum_{k=1}^{K_s} \log_2(\text{Num}_k + \text{Den}_k)$, $f_2(\mathbf{p}_r) = \sum_{k=1}^{K_s} \log_2(\text{Den}_k)$, and Num_k and Den_k are the numerator and denominator of $\Gamma_k^d(\mathbf{p}_r)$ in (14). We note that $f_1(\mathbf{p}_r)$ and $f_2(\mathbf{p}_r)$ are concave functions. However, $f_1(\mathbf{p}_r) - f_2(\mathbf{p}_r)$ is not concave. To obtain a lower bound that is concave, we upper bound $f_2(\mathbf{p}_r)$ with its first order Taylor's series expansion $\bar{f}_2\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right)$ around $\bar{\mathbf{p}}_r^0 = \left[\bar{p}_{1r}^0, \bar{p}_{2r}^0, \ldots, \bar{p}_{K_s r}^0\right]^T$. It is given by $\bar{f}_2\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right) = f_2\left(\bar{\mathbf{p}}_r^0\right) + \nabla f_2\left(\bar{\mathbf{p}}_r^0\right)^T\left(\mathbf{p}_r - \bar{\mathbf{p}}_r^0\right)$. Using this, we get the following lower bound,

$$\sum_{k=1}^{K_s} \log_2\left(1 + \Gamma_k^d(\mathbf{p}_r)\right) \geq f_1(\mathbf{p}_r) - \bar{f}_2\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right). \tag{18}$$

Substituting the above inequality in (15) yields $\text{EE}_{\text{lb}}\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right)$, which is a lower bound on the objective of the distributed problem

$$\text{EE}_{\text{lb}}\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right) = \frac{f_1(\mathbf{p}_r) - \bar{f}_2\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right)}{\eta \sum_{i=1}^{K_s} p_{ir} + \eta(L-1)K_s P_{eq} + \text{C}_{\text{p}}}. \tag{19}$$

For a given $\bar{\mathbf{p}}_r^0$, with (19) as the objective function, we formulate the following optimization problem

$$\begin{aligned} & \max_{\mathbf{p}_r} \quad \text{EE}_{\text{lb}}\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right), \\ & \text{s.t.} \quad (9),\ (10),\ p_{ir} \geq 0, i = 1, \ldots, K_s, \end{aligned} \tag{20}$$

$$\Gamma_k^d\left(\mathbf{p}_r\right)=\frac{\text{Num}_k}{\text{Den}_k}=\frac{\left(\sqrt{p_{kr}\,\text{Trace}\left(\widehat{\mathbf{R}}_{kl}\right)}+\sqrt{P_{\text{eq}}}\sum_{l=1,l\neq r}^{L}\sqrt{\text{Trace}\left(\widehat{\mathbf{R}}_{kl}\right)}\right)^2}{\sum_{i=1}^{K_s}p_{kr}n_{ikr}+P_{\text{eq}}\sum_{l=1,l\neq r}^{L}\sum_{i=1}^{Ks}(n_{ikl})+\sum_{i=1,i\neq k}^{K_s}\left(\sqrt{p_{ir}}m_{ikr}+\sqrt{P_{eq}}\sum_{l=1,l\neq r}^{L}m_{ikl}\right)^2+\sigma_k^2}. \quad (14)$$

which has a fractional objective with concave function in the numerator and linear function in the denominator.

Iterative Algorithm: Dinkelbach's algorithm [12], which optimizes the difference between the numerator and scaled denominator of the fractional objective function, can solve the above fractional problem. For the fractional objective in (19), we define the function $F(\mathbf{p}_r, \bar{\mathbf{p}}_r^0, \lambda_n)$ optimized by the Dinkelbach's algorithm. It is a function of $\mathbf{p}_r$, the vector $\bar{\mathbf{p}}_r^0$ around which we expand $f_2(\mathbf{p}_r)$, and Dinkelbach's iterative parameter λ_n given by

$$F(\mathbf{p}_r, \bar{\mathbf{p}}_r^0, \lambda_n) = f_1(\mathbf{p}_r) - \bar{f}_2\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^0\right) - \lambda_n\left(\eta(L-1)K_sP_{eq} + \eta\sum_{i=1}^{K_s}p_{ir} + \text{C}_\text{p}\right).$$

Therefore, at r^{th} AP, the optimization problem for the n^{th} iteration of the Dinkelbach's algorithm is given by

$$\mathcal{P}_n^r : \max_{\mathbf{p}_r} \quad F(\mathbf{p}_r, \bar{\mathbf{p}}_r^0, \lambda_n), \quad (21a)$$

$$\text{s.t.} \quad (9),\ (10),\ p_{ir} \geq 0, i = 1, \ldots, K_s. \quad (21b)$$

The Dinkelbach's parameter for the $(n+1)^{\text{th}}$ iteration is obtained by substituting the solution of $\mathcal{P}_n^r$ in (19). Once the function $F(\mathbf{p}_r, \bar{\mathbf{p}}_r^0, \lambda_n)$ converges for a given $\bar{\mathbf{p}}_r^0$, we repeat the process by expanding $f_2(\mathbf{p}_r)$ around the solution obtained. This process is repeated till the objective in (19) converges.

To further improve the performance of the above iterative algorithm, we perform scaling and to reduce complexity, we utilize distributed equal power. *i) Scaling:* The solution obtained by the above iterative algorithm can be conservative due to the tight interference threshold I_{th}/L in (10). Therefore, for small $P_{\max}$ values, we scale the solution obtained by solving $\mathcal{P}_n^r$ to meet the power constraint in (9) while ensuring the interference constraint is not violated. *ii) Distributed Equal Power $P_{eq}^d(r)$:* Substituting $p_{ir} = P_{\text{eq}}^d(r)$ in (9) and (10), we get the following equal power that satisfies both these constraints with equality

$$P_{\text{eq}}^d(r) = \min\left\{P_{\max}/K_\text{s}, I_{\text{th}}/LB_r\right\}, \quad (22)$$

where $B_r = \max_m\{\sum_{i=1}^{K_s} b_{imr}\}$. This equal power differs from P_{eq} in (11) and is different for each AP.

For smaller values of $P_{\max}$, i.e., if $P_{\max} \leq \frac{K_\text{s}I_{\text{th}}}{LB_r}$, we use the scaled version of the solution obtained from the above iterative algorithm. Otherwise, the r^{th} AP allocates $p_{ir} = \frac{I_{\text{th}}}{LB_r}$ for $i = 1, \ldots, K_\text{s}$. The pseudo-code of our proposed distributed algorithm is described in Algorithm 1. It combines Dinkelbach's algorithm, power scaling, and use of distributed equal power to improve performance and reduce complexity. The complete power allocation vector $\mathbf{p} = [\mathbf{p}_1^T, \mathbf{p}_2^T, \ldots, \mathbf{p}_L^T]$. Substituting this in (8) gives us the EE of the complete system.

For a given initial point $\bar{\mathbf{p}}_r^t$, the Dinkelbach's algorithm in the inner loop of Algorithm 1 converges to the global solution of (20) in a super-linear convergence rate [12]. Furthermore, the use of convex lower bound ensures that the outer loop converges to a local maximum of the problem in (16).

Unlike the centralized approach, where one optimization algorithm is solved centrally at the CPU, now each AP runs the above algorithm locally. This distributed algorithm at each AP solves only for K_s variables unlike in the centralized approach where the CPU solves for LK_s variables jointly.

Algorithm 1 Distributed optimal power allocation at r^{th} AP

Input: $P_{\max}$, I_{th}, and required channel statistics
if $P_{\max} \leq \frac{K_\text{s}I_{\text{th}}}{LB_r}$ **then**
  Set $t = 0$ and choose a feasible initial point $\bar{\mathbf{p}}_r^t$
  while $\text{EE}_{\text{lb}}\left(\mathbf{p}_r, \bar{\mathbf{p}}_r^t\right)$ does not converge **do**
    Initialize $\epsilon > 0$; $n = 0; \lambda_0 = 0$;
    do
      Solve $\mathcal{P}_n^r$ in (21) and obtain the solution $\mathbf{p}_{rn}^*$;
      $\lambda_{n+1} = \text{EE}_{\text{lb}}\left(\mathbf{p}_{rn}^*, \bar{\mathbf{p}}_r^t\right)$;
      $n = n + 1$;
    while $F(\mathbf{p}_{r(n-1)}^*, \bar{\mathbf{p}}_r^t, \lambda_n) > \epsilon$
    Set $t = t + 1$;
    Set $\bar{\mathbf{p}}_r^t = \mathbf{p}_{r(n-1)}^*$;
  end while
  Scale $\mathbf{p}_{rn}^*$ to meet the constraints (9) and (10).
else
  $p_{ir} = \frac{I_{\text{th}}}{LB_r}$, for $i = 1, \ldots, K_s$.
end if

IV. Numerical Results

In this Section, we study and benchmark the performance of the proposed distributed optimal power allocation algorithm. In our outdoor primary system, the users are uniformly distributed over a 100 m by 100 m area. The secondary APs are in an area of 125 m by 125 m room, and secondary users are uniformly distributed inside the room. We consider a simplified pathloss model with a pathloss exponent of 3.7 and thermal noise with $\sigma^2 = -92$ dBm. Local scattering spatial correlation model is considered for the channel gains. EE averaged over 100 simulation setups of uniformly distributed users are shown. We set $B = 20$ MHz, $\tau_\text{c} = 2000$, and $\tau_\text{p} = 8$. All results are shown for six secondary APs with four antennas each, serving four secondary users and shares spectrum with five antenna BS serving four primary users.

Figure 2 compares the performance of the proposed distributed algorithm with the centralized algorithm proposed in [9]. EE as a function of $P_{\max}$ is shown. For small values of $P_{\max}$, the distributed approach matches with the centralized

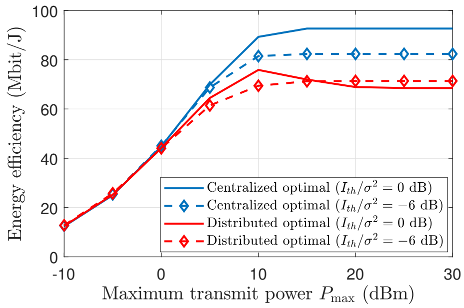
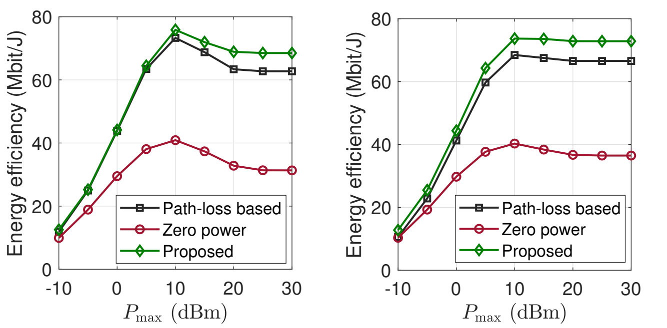


Fig. 2. Comparison with centralized approach: EE versus $P_{\max}$ for different values of I_{th}/σ^2.



(a) $I_{\text{th}}/\sigma^2 = 0$ dB (b) $I_{\text{th}}/\sigma^2 = -3$ dB

Fig. 3. Comparison of distributed power allocation algorithms: EE as a function of $P_{\max}$.

approach. Here, the transmit power constraint is active and the average interference at the primary user is below I_{th}. Hence, the EE is limited by the $P_{\max}$ and it increases as $P_{\max}$ increases. We refer to it as power constrained regime. For large values of $P_{\max}$, the interference constraint becomes active. Here, the I_{th} limits the total transmit power, which is lower than the $P_{\max}$. We refer to this as interference constrained regime. Here, for $I_{\text{th}}/\sigma^2 = 0$ dB, after $P_{\max} = 10$ dBm, the EE decreases and then saturates for distributed approach. However, it does not decrease for the centralized approach due to joint optimization at the CPU. For $I_{\text{th}}/\sigma^2 = -6$ dB, EE saturates for both approaches. At $P_{\max} = 10$ dBm, the distributed approach achieves 85% EE of the centralized approach for $I_{\text{th}}/\sigma^2 = 0$ dB and $I_{\text{th}}/\sigma^2 = -6$ dB. For $P_{\max} \geq 20$ dBm, distributed optimal uses equal power in (22), which allows higher power for higher I_{th} and yields lower EE.

We now benchmark the proposed distributed optimal algorithm with other distributed approaches.

Zero-Power Approach: Here, the r^{th} secondary AP formulates its local optimization problem by substituting zeros for the power allocation coefficients of other APs. It substitutes, $p_{il} = 0, l \neq r, i = 1, \ldots, K_s$ in (8), and obtains p_{ir} that maximizes it subject to the constraints using the convex lower bound approach described in Section III-A. Finally, all the APs transmit with the K_s power allocation variables solved locally.

Path-Loss Based Approach [1, Sec. 7.2.3]: Let $\beta_{ir} = \text{Trace}\left(\mathbf{R}_{ir}\right)/N$. For the k^{th} secondary user, the AP r allocates $p_{kr} = P_{\max}\left(\beta_{kr}\right)^{0.5} / \left(\sum_{i=1}^{K_s}\left(\beta_{ir}\right)^{0.5}\right)$, which satisfies the transmit power constraint. For a fair comparison, we scale the above power allocation to satisfy the interference constraint.

Fig. 3 compares the performance of the distributed optimal algorithm with the above two distributed approaches. For small $P_{\max}$, the performance of the scalable distributed approach matches with the distributed optimal. However, for large $P_{\max}$, the distributed optimal performs better. At $P_{\max} = 30$ dBm, it yields 12% and 9% higher EE than the scalable distributed approach for $I_{\text{th}}/\sigma^2 = 0$ dB and $I_{\text{th}}/\sigma^2 = -3$ dB, respectively. We also see that the distributed optimal performs significantly better than the single AP approach for all $P_{\max}$ values. At $P_{\max} = 30$ dBm, it yields $2.2\times$ and $2\times$ higher EE than the single AP approach for $I_{\text{th}}/\sigma^2 = 0$ dB and $I_{\text{th}}/\sigma^2 = -3$ dB, respectively.

V. Conclusion

We proposed a distributed downlink power allocation algorithm, which improved EE of a secondary cell-free system. It reduced computational complexity at the CPU and allowed parallel computation locally at each secondary AP. We showed that its performance matched with the centralized approach in the power-constrained regime with small $P_{\max}$. Furthermore, we demonstrated that tight interference-constraints can lead to decrease in the EE as $P_{\max}$ increases. Our performance benchmarking showed that the proposed algorithm performed better than the simpler path-loss based power allocation and the zero power approach.

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