Simultaneously Transmitting And Reflecting (STAR) IRS-Aided OTFS System

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***Abstract*—We develop a novel and comprehensive model for a simultaneously transmitting and reflecting (STAR) intelligent reflecting surface (IRS)-aided orthogonal time frequency space (OTFS) system. We propose two novel algorithms, namely, equal split STAR IRS phase programming and two step amplitude and phase programming, to reconfigure both the transmission and reflection coefficient of the passive elements in an STAR IRS-aided OTFS system and also analyze their computational complexities. We show that the second algorithm performs noticeably better than the first at marginally higher complexity. We also elucidate that an STAR IRS outperforms a conventional IRS in an OTFS system as the number of passive elements increases. Our work also suggests that 3-bit discrete phase shifts at the STAR IRS can achieve the same BER performance as an STAR IRS with infinite phase precision.**

***Index Terms*—OTFS modulation, Intelligent Reflecting Surface (IRS), Simultaneously Transmitting And Reflecting (STAR) IRS.**

I. Introduction

High mobility applications must be supported in the sixth generation (6G) wireless network architecture. However, under high mobility, high Doppler shift causes interference among subcarriers in the existing orthogonal frequency division multiplexing (OFDM) systems and adversely impacts the reliability of data detection [1], [2]. In order to improve resilience to large Doppler shifts in 6G, orthogonal time frequency space (OTFS) modulation can be a viable alternative. OTFS exploits the sparse and time-invariant characteristics of wireless channels to send data in the delay-Doppler (DD) domain [3]. Another possible 6G technology that is being considered is intelligent reflecting surface (IRS) [4], [5]. It is made up of several inexpensive reflecting components that do not need dedicated radio frequency (RF) chains. Additionally, constructive superposition of the signals that reach the receiver may be achieved by suitably programming phase shifts caused by these passive elements, maximising the power of the received signal. Thus, IRS along with OTFS can improve data rate, energy economy and data detection reliability in high-Doppler applications anticipated in 6G systems.

Existing literature mostly consider the case where the IRSs are only capable of reflecting the incident waves. Throughout this paper, we refer to such reflect-only IRSs as conventional IRSs. This imposes a geographical restriction that both the transmitter and receiver have to be located on the same side of the IRS limiting the smart radio environment (SRE) to half. In reality, this constraint may not always be satisfied which, in turn, limits the flexibility and efficacy of the conventional IRSs. The innovative idea of simultaneously transmitting and reflecting (STAR) IRSs for a non-OTFS system was put out in [6] in order to avoid this restriction. Specifically, a wireless signal incident on an element of an STAR IRS is split into two halves [6]. The transmitted signal, which is sent to the space opposite to the incident signal, and the reflected signal, which is reflected to the same space as the incident signal. And the corresponding spaces are referred to as the transmission (T) and reflection (R) spaces, respectively.

A. Motivation and Contributions

While in [7], the relation between input and output was analyzed for a conventional IRS-enabled OTFS system considering fractional DD values and rectangular waveforms. The application of IRS-aided OTFS systems in space-air ground integrated networks was proposed and its performance under channel estimation was analyzed in [8]. System modeling, optimization and performance analysis of an STAR IRS system was investigated in [9] and the three different working protocols of STAR IRS was discussed in detail. Algorithms for antenna selection and phase shift optimization in an IRS-aided OTFS system was presented in [10].

To the best of our knowledge, there is no literature available on system modelling, phase optimisation, and performance analysis of STAR IRS-aided OTFS system for high Doppler scenarios, where the passive elements use more degrees of freedom (DoF) and data is multiplexed in the DD domain. Also, it is significantly more challenging to configure elements of an

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STAR IRS since they can both reflect and transmit at the same time. To this end, we make the following novel contributions:

- We develop a novel model for an STAR IRS-aided OTFS system in which the transmitter communicates with two receivers located on either side of the IRS, i.e, one in the T-space and another in the R-space.
- We propose two novel algorithms for selecting the reflection and transmission coefficients of the STAR IRS elements. In the first algorithm, we propose to split the incident signal power equally between the transmitted and reflected signals and then optimize the phase shifts corresponding to both the transmission and reflection coefficients. In the second algorithm, we also optimize the magnitude of these coefficients along with the phase shifts in order to achieve better reliability of data detection. The computational complexities of the two algorithms are also analyzed.
- Through our numerical results, we infer that for lower number of IRS elements, conventional IRS may give identical performance as STAR IRS in an OTFS system. However, as we increase the number of elements, the STAR IRS begins to show its effectiveness over conventional IRS even at high Doppler. We also elucidate that a 3-bit phase shifter at the STAR IRS may be sufficient to achieve same BER performance as an STAR IRS with infinite phase precision.

II. System Model

We consider an STAR IRS-aided OTFS system in which the L passive elements at the IRS operate in the energy splitting (ES) mode[1] and assist a single antenna transmitter in communicating with two receivers as shown in Figure 1. We consider that one of the receivers is located in the R-space of the STAR IRS while the other receiver is located in the T-space. The direct paths between the transmitter and the receivers are considered to be blocked. Let $x[k,l]$ denote the DD domain information symbol sent from the transmit antenna in a two dimensional $M \times N$ bin represented by $\mathcal{S} = \left\{\left(\frac{k}{NT}, \frac{l}{M\Delta f}\right), k = 0,\ldots,N-1, l = 0,\ldots,M-1\right\}$, where $\frac{1}{NT}$ and $\frac{1}{M\Delta f}$ denote the Doppler and delay resolutions, respectively. Furthermore, N and M represent the total number of Doppler and delay taps, respectively. The channel impulse response from the transmitter to the i^{th} element of STAR IRS in DD domain is given by [11]

$$g^i(\tau,\nu) = \sum_{p=1}^{P} g_p^i \delta(\tau - \tau_p^i)\delta(\nu - \nu_p^i), \tag{1}$$

where $i = 1,\ldots,L$. In (1), P denotes the total number of paths from the transmit antenna to the i^{th} element of STAR IRS. Here τ_p^i and ν_p^i denote the delay and the Doppler shifts, respectively, along path p and g_p^i is the channel gain from transmitter to i^{th} element along path p and we consider that $g_p^i \sim \mathcal{CN}\left(0, \frac{1}{P}\right)$. The channel impulse response from the i^{th} STAR IRS element to the receiver in R-space can be expressed similarly in the DD domain as

$$h_r^i(\tau,\nu) = \sum_{q_r=1}^{Q_r} h_{q_r}^i \delta(\tau - \tau_{q_r}^i)\delta(\nu - \nu_{q_r}^i), \tag{2}$$

where Q_r denotes total number of paths from the i^{th} element of STAR IRS to the receiver located in R-space, $\tau_{q_r}^i$ and $\nu_{q_r}^i$ denote the delay and the Doppler shift associated with path q_r and $h_{q_r}^i$ denotes the channel gain from the i^{th} element to the receiver in R-space along path q_r and it is assumed to follow $\mathcal{CN}\left(0, \frac{1}{Q_r}\right)$ distribution. Similarly, the channel impulse response from the i^{th} STAR IRS element to the receiver in T-space can be expressed in the DD domain as

$$h_t^i(\tau,\nu) = \sum_{q_t=1}^{Q_t} h_{q_t}^i \delta(\tau - \tau_{q_t}^i)\delta(\nu - \nu_{q_t}^i), \tag{3}$$

where Q_t denotes total number of paths from the i^{th} element of STAR IRS to the receiver in T-space, $\tau_{q_t}^i$ and $\nu_{q_t}^i$ denote the delay and the Doppler shift associated with path q_t and $h_{q_t}^i$ denotes the channel gain from the i^{th} element to the receiver in T-space along path q_t and we consider that it follows $\mathcal{CN}(0, \frac{1}{Q_t})$ distribution. For ease of exposition, we assume that $Q_r = Q_t = Q$. Let the reflection coefficient of i^{th} element be ϕ_i^r and the corresponding transmission coefficient be denoted by ϕ_i^t. Let γ_i^r and θ_i^r denote the amplitude and phase of the reflection coefficient and γ_i^t and θ_i^t denote the amplitude and phase of the transmission coefficient, respectively. Then, ϕ_i^r and ϕ_i^t can be written as $\phi_i^r = \gamma_i^r\, e^{j\theta_i^r}$ and $\phi_i^t = \gamma_i^t\, e^{j\theta_i^t}$, where $\gamma_i^r, \gamma_i^t \in (0,1)$, ${\gamma_i^r}^2 + {\gamma_i^t}^2 = 1$ and $\theta_i^r, \theta_i^t \in [-\pi, \pi]$. Then, the DD domain symbol $y_r^i[k,l]$ received at the R-space user via the i^{th} STAR IRS element is given by [11]

$$y_r^i[k,l] = e^{j\theta_i^r} \sum_{q_r=1}^{Q}\sum_{p=1}^{P} h_{pq_r}^i e^{-j2\pi\nu_{pq_r}^i \tau_{pq_r}^i} \times x[[k-\beta_{pq_r}^i]_N, [l-\alpha_{pq_r}^i]_M] + v_r^i[k,l], \tag{4}$$

where $\nu_p^i + \nu_{q_r}^i = \nu_{pq_r}^i$, $\tau_p^i + \tau_{q_r}^i = \tau_{pq_r}^i$, $\rho_{pq_r}^i = e^{-j2\pi\nu_{q_r}^i \tau_p^i}$ and $g_p^i \times h_{q_r}^i \times \rho_{pq_r}^i = h_{pq_r}^i$. We assume $\alpha_{pq_r}^i$ and $\beta_{pq_r}^i$ to be integral multiples of delay and Doppler resolutions, respectively [7]. The term $v_r^i[k,l]$ denotes the corresponding additive white Gaussian noise (AWGN) in DD domain. The DD domain symbol $y_t^i[k,l]$ received at the T-space user via the i^{th} STAR IRS element is given by

$$y_t^i[k,l] = e^{j\theta_i^t} \sum_{q_t=1}^{Q}\sum_{p=1}^{P} h_{pq_t}^i e^{-j2\pi\nu_{pq_t}^i \tau_{pq_t}^i} \times x[[k-\beta_{pq_t}^i]_N, [l-\alpha_{pq_t}^i]_M] + v_t^i[k,l], \tag{5}$$

[1] All STAR IRS elements are assumed to simultaneously transmit and reflect based on ES, with an ES ratio of γ_i^r: γ_i^t. This suggests that the energy of the signal incident on each element is typically divided into the energies of the transmitted and the reflected signals according to the specified ratio.

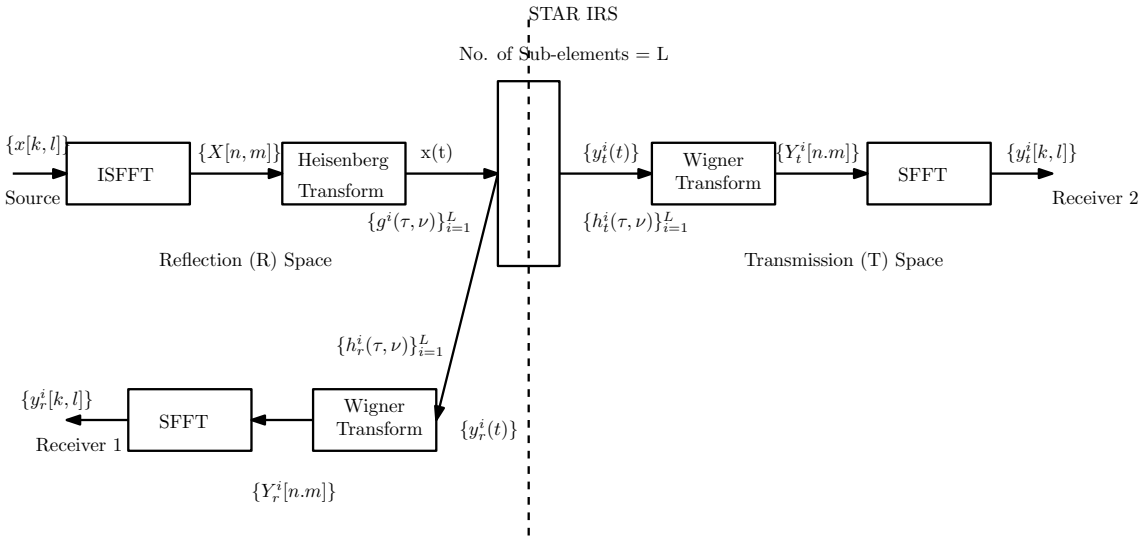


Fig. 1: System Model for an STAR IRS-Aided OTFS System

where $\nu^i_{pq_t} = \nu^i_p + \nu^i_{q_t}$, $\tau^i_{pq_t} = \tau^i_p + \tau^i_{q_t}$, $\rho^i_{pq_t} = e^{-j2\pi\nu^i_{q_t}\tau^i_p}$, $g^i_p \times h^i_{q_t} \times \rho^i_{pq_t} = h^i_{pq_t}$, $\alpha^i_{pq_t}$ and $\beta^i_{pq_t}$ are assumed to be integral multiples of delay and Doppler resolutions, respectively. Now, (4) and (5) can be expressed in a vectorized form as

$$\boldsymbol{y_r^i} = \phi^r_i \boldsymbol{H^i_r x} + \boldsymbol{v^i_r}, \;\; \boldsymbol{y^i_t} = \phi^t_i \boldsymbol{H^i_t x} + \boldsymbol{v^i_t}, \tag{6}$$

where $\boldsymbol{y^i_r}$, $\boldsymbol{y^i_t}$, $\boldsymbol{v^i_r}$ and $\boldsymbol{v^i_t} \in C^{MN\times 1}$, $\boldsymbol{x} \in C^{MN\times 1}$, $\boldsymbol{H^i_r}$ and $\boldsymbol{H^i_t} \in C^{MN\times MN}$ represent the cascaded channel-matrix from the transmitter to the receiver in the R and T-spaces, respectively, through the i^{th} element of the IRS. Summing up signals coming from all the L elements at the receiver, we get DD domain input-output relation as $\boldsymbol{y_r} = \sum_{i=1}^L \phi^r_i \boldsymbol{H^i_r x} + \boldsymbol{v_r} = \boldsymbol{H_r x} + \boldsymbol{v_r}$ and $\boldsymbol{y_t} = \sum_{i=1}^L \phi^t_i \boldsymbol{H^i_t x} + \boldsymbol{v_t} = \boldsymbol{H_t x} + \boldsymbol{v_t}$, where $\sum_{i=1}^L \phi^r_i \boldsymbol{H^i_r} = \boldsymbol{H_r}$, $\sum_{i=1}^L \phi^t_i \boldsymbol{H^i_t} = \boldsymbol{H_t}$ and $\boldsymbol{y_r}$, $\boldsymbol{y_t} \in \mathcal{C}^{MN\times 1}$ denote the combined received vector from all the elements corresponding to the two different receivers, and $\boldsymbol{v}_r$ and $\boldsymbol{v}_t \in \mathcal{C}^{MN\times 1}$ denote the noise vectors at the corresponding receivers.

III. Proposed Algorithms for Magnitude and Phase Optimization for STAR IRS

In a conventional reflect-only IRS-aided OTFS system, we assume that the wireless signal incident on the IRS elements would be fully reflected. Hence, in such systems, only the magnitude and phase of the reflection coefficient needs to be optimized. The major challenge involved in designing the STAR IRS-aided OTFS system is to not only optimise the phase shift and magnitude corresponding to user in R-space but also optimize the magnitude and phase corresponding to user in T-space. With the optimal values of reflection and transmission coefficient, the Forbenius norm of the overall effective channel matrix corresponding to transmission and reflection modes should be maximized. In this section, we first consider the incident power to be equally divided between the transmitted and reflected signals. Thereafter, we propose a novel magnitude and phase optimisation algorithm for STAR IRS aided OTFS system. In these two algorithms, $[j] = \max(\boldsymbol{b})$ returns the index j of the largest element in the vector $\boldsymbol{b}$.

A. Equal Split STAR IRS Phase Programming

In this method, for each of the elements of IRS, we assume that the incident signal power gets equally distributed between the transmitted and reflected signals. This implies that $(\gamma^r_i)^2 = (\gamma^t_i)^2 = 0.5$. Hence, the reflection and transmission coefficient vectors are $\boldsymbol{\Phi}_r = \left[\sqrt{0.5}e^{j\theta^r_1} \sqrt{0.5}e^{j\theta^r_2} \ldots \sqrt{0.5}e^{j\theta^r_L}\right]$ and $\boldsymbol{\Phi}_t = \left[\sqrt{0.5}e^{j\theta^t_1} \sqrt{0.5}e^{j\theta^t_2} \ldots \sqrt{0.5}e^{j\theta^t_L}\right]$, respectively. Next, we generate a large number of L-length random reflection and transmission coefficient vectors where phases induced by each element is uniformly distributed in $[-\pi, \pi]$ [7]. Let us assume that the total number of such randomly generated phase vectors is U. The u^{th} realization of the reflection and transmission coefficient vectors are denoted by $\boldsymbol{\Phi}^u_r = \left[\sqrt{0.5}e^{j\theta^r_{1u}} \sqrt{0.5}e^{j\theta^r_{2u}} \ldots \sqrt{0.5}e^{j\theta^r_{Lu}}\right]$ and $\boldsymbol{\Phi}^u_t = \left[\sqrt{0.5}e^{j\theta^t_{1u}} \sqrt{0.5}e^{j\theta^t_{2u}} \ldots \sqrt{0.5}e^{j\theta^t_{Lu}}\right]$. Furthermore, the channel matrix from the transmit antenna to the receiver in the R-space for the u^{th} realization is given by $\boldsymbol{H}_r(u) = \sum_{i=1}^L \sqrt{0.5}e^{j\theta^r_{iu}} \boldsymbol{H^i_r}$. Similarly, the channel matrix from the transmit antenna to the receiver in the T-space for the u^{th} realization is given by $\boldsymbol{H}_t(u) = \sum_{i=1}^L \sqrt{0.5}e^{j\theta^t_{iu}} \boldsymbol{H^i_t}$. Next, we select that particular transmission and reflection coefficient vector among the U realizations to be the optimum one which gives us maximum Forbenius norm of the effective channel matrix from the transmitter to the receiver in the T-space and R-space, respectively. We find out the corresponding optimal transmission and reflection coefficients from the randomly generated coefficient vectors independently. Mathematically, the selected reflection and transmission coefficient, denoted by

Algorithm 1: Equal Split STAR IRS Phase Programming

Input: $\boldsymbol{H}_r^i$, $\boldsymbol{H}_t^i$ where i = 1,2,...,L and U random realizations of the L-length reflection and transmission coefficient vectors.
Output: $\boldsymbol{\Phi}_{\boldsymbol{r}}^*$ and $\boldsymbol{\Phi}_{\boldsymbol{t}}^*$.
Generate: $\boldsymbol{\Theta}_u$= $[\theta_{1u}\ \ \theta_{2u}\ \ \ldots\ \ \theta_{Lu}]$, where $\theta_{iu} \in \mathcal{U}[-\pi, \pi]$; i= 1, 2,..., L.
$\boldsymbol{\Phi}_u \leftarrow [\sqrt{0.5}\theta_{1u}\ \ \sqrt{0.5}\theta_{2u}\ \ \ldots\ \ \sqrt{0.5}\theta_{Lu}]$
Initialize $u_r^* = 0$, $\psi_r^* = 0$, $u_t^* = 0$ and $\psi_t^* = 0$;
for $u = 1, u \leq U, u++$ **do**
  $\boldsymbol{H}_r \leftarrow \sum_{i=1}^{L} \sqrt{0.5} e^{j\theta_{iu}} \boldsymbol{H}_r^i$ and $\boldsymbol{H}_t \leftarrow \sum_{i=1}^{L} \sqrt{0.5} e^{j\theta_{iu}} \boldsymbol{H}_t^i$
  $\boldsymbol{\psi}_r[u] \leftarrow ||\boldsymbol{H}_r||^2$ and $\boldsymbol{\psi}_t[u] \leftarrow ||\boldsymbol{H}_t||^2$;
u_r^* = max($\boldsymbol{\psi}_r$) and u_t^* = max($\boldsymbol{\psi}_t$);
return $\boldsymbol{\Phi}_{\boldsymbol{r}}^* = \boldsymbol{\Phi}_{u_r^*}$ and $\boldsymbol{\Phi}_{\boldsymbol{t}}^* = \boldsymbol{\Phi}_{u_t^*}$.

u_r^* and u_t^*, are given by

$$u_r^* = \underset{u}{\text{argmax}}\ ||\boldsymbol{H}_r(u)||^2, \quad u_t^* = \underset{u}{\text{argmax}}\ ||\boldsymbol{H}_t(u)||^2, \quad (7)$$

where $u \in \{1, 2, \ldots, U\}$ and the corresponding optimal reflection and transmission coefficients are $\boldsymbol{\Phi}_{\boldsymbol{r}}^* = \boldsymbol{\Phi}_{u_r^*}^r$ and $\boldsymbol{\Phi}_{\boldsymbol{t}}^* = \boldsymbol{\Phi}_{u_t^*}^t$. The above procedure is summarised in Algorithm 1. The computational complexity of this algorithm depends on the complexity of max(·) function that has been used in the algorithm. The complexity associated with finding the index of the largest element in any N length vector using max function that we have used is $\mathcal{O}(N)$. Time complexity calculation for this algorithm can be explained based on the following steps:

- For each of the randomly generated phase vectors, computation of $\boldsymbol{H}_r$ and $\boldsymbol{H}_t$ involves L complex additions. So, the time complexity is given by $\mathcal{O}(UL)$.
- For each of the randomly generated phase vectors, to get the optimal transmission and reflection coefficient, we need to find u_r^* and u_t^* by taking max of $\boldsymbol{\psi}_r$ and $\boldsymbol{\psi}_t$ respectively, which are U length vectors. So, the time complexity becomes $\mathcal{O}(U)$.

Hence, the overall complexity for the algorithm will be the sum of the time complexities of all these steps. In the algorithm above, we did not optimize the amplitudes corresponding to the transmission and reflection modes. We instead assumed that the incident signal power gets equally distributed between the transmitted and the reflected signals, which may not always be true. To this end, we next propose a two step amplitude and phase programming in the STAR IRS-aided OTFS system.

B. Two Step Amplitude and Phase Programming in STAR IRS

In two step amplitude and phase programming of STAR IRS-aided OTFS system, for each elements, we generate a large number of L-length random reflection and transmission coefficient vectors where the phase induced by each element is uniformly distributed in $[-\pi, \pi]$ and their amplitude is also assumed to be uniformly distributed between $[0, 1]$. Let us assume, we first generate the reflection coefficient and the total number of such randomly generated reflection coefficients is U. The u^{th} realization of the reflection coefficient vector is $\boldsymbol{\Phi}_r^u = \left[\gamma_{1u}^r e^{j\theta_{1u}^r}\ \gamma_{2u}^r e^{j\theta_{2u}^r} \ldots \gamma_{Lu}^r e^{j\theta_{Lu}^r}\right]$, where $\boldsymbol{\Gamma}_{\boldsymbol{u}}^{\boldsymbol{r}} = [\gamma_{1u}^r\ \gamma_{2u}^r \ldots \gamma_{Lu}^r]$ is the corresponding magnitude vector and $\boldsymbol{\Theta}_{\boldsymbol{u}}^{\boldsymbol{r}} = [\theta_{1u}^r\ \theta_{2u}^r \ldots \theta_{Lu}^R]$ is the phase vector. Here, $\gamma_{iu}^r \in \mathcal{U}(0, 1)$ and $\theta_{iu}^r \in \mathcal{U}[-\pi, \pi]$, for all $i = 1, 2, \ldots, L$. The channel matrix from the transmit antenna to the receiver in the R-space for the u^{th} realization is given by $\boldsymbol{H}_r(u) = \sum_{i=1}^{L} \gamma_{iu}^r e^{j\theta_{iu}^r} \boldsymbol{H}_r^i$. Next, we select that reflection coefficient vector among the U realizations to be the optimum one, which gives us maximum Forbenius norm of the effective channel matrix from the transmitter to the receiver in the R-space. Mathematically, the selected reflection coefficient, denoted by u_r^* is given by $u_r^* = \text{argmax}_u\ ||\boldsymbol{H}_r(u)||^2$, where $u \in \{1, 2, \ldots, U\}$ and the corresponding STAR IRS reflection coefficient is $\boldsymbol{\Phi}_{\boldsymbol{r}}^* = \boldsymbol{\Phi}_{u_r^*}^r$. The corresponding optimal magnitude vector for reflection coefficient is given as $\boldsymbol{\Gamma}_r^* = \boldsymbol{\Gamma}_{u_r^*}^r$. We next compute the optimal magnitude vector for transmission coefficient $\boldsymbol{\Gamma}_t^*$ as $\boldsymbol{\Gamma}_t^* = [\gamma_1^{t^*}\ \gamma_2^{t^*}\ \ldots\ \gamma_L^{t^*}]$ where $\gamma_i^{t^*} = \sqrt{1 - \left(\gamma_{iu_r^*}^r\right)^2}$. The u^{th} realization of the reflection coefficient vector is $\boldsymbol{\Phi}_t^u = \left[\gamma_1^{t^*} e^{j\theta_{1u}}\ \gamma_2^{t^*} e^{j\theta_{2u}} \ldots \gamma_L^{t^*} e^{j\theta_{Lu}}\right]$ and the channel matrix from

Algorithm 2: Two Step Amplitude and Phase Programming in STAR IRS

Input: $\boldsymbol{H}_r^i$, $\boldsymbol{H}_t^i$ where i = 1,2,...,L and U random realizations of the L-length reflection and transmission coefficient vectors.
Output: $\boldsymbol{\Phi}_{\boldsymbol{r}}^*$ and $\boldsymbol{\Phi}_{\boldsymbol{t}}^*$.
Generate: $\boldsymbol{\Theta}_u$= $[\theta_{1u}\ \ \theta_{2u}\ \ \ldots\ \ \theta_{Lu}]$ and $\boldsymbol{\Gamma}_u^r$= $[\gamma_{1u}^r\ \ \gamma_{2u}^r\ \ \ldots\ \ \gamma_{Lu}^r]$ where $\theta_{iu}^r \in \mathcal{U}[-\pi, \pi]$ and $\gamma_{iu}^r \in \mathcal{U}(0, 1)$; $i = 1, 2, \ldots, L$.
$\boldsymbol{\Phi}_u^r \leftarrow [\gamma_{iu}^r e^{j\theta_{1u}}\ \ \gamma_{iu}^r e^{j\theta_{2u}}\ \ \ldots\ \ \gamma_{iu}^r e^{j\theta_{Lu}}]$
Initialize $u_r^* = 0$, $\psi_r^* = 0$;
for $u = 1, u \leq U, u++$ **do**
  $\boldsymbol{H}_r \leftarrow \sum_{i=1}^{L} \gamma_{iu}^r e^{j\theta_{iu}} \boldsymbol{H}_r^i$
  $\boldsymbol{\psi}_r[u] \leftarrow ||\boldsymbol{H}_r||^2$;
u_r^* = max($\boldsymbol{\psi}_r$);
$\boldsymbol{\Gamma}_t^* \leftarrow [\gamma_1^{t^*}\ \gamma_2^{t^*}\ \ldots\ \gamma_L^{t^*}]$, where $\gamma_i^{t^*} = \sqrt{1 - \gamma_{iu_r^*}^{r\ 2}}$
$\boldsymbol{\Phi}_u^t \leftarrow [\gamma_1^{t^*} e^{j\theta_{1u}}\ \ \gamma_2^{t^*} e^{j\theta_{2u}}\ \ \ldots\ \ \gamma_L^{t^*} e^{j\theta_{Lu}}]$
Initialize $u_t^* = 0$, $\psi_t^* = 0$;
for $u = 1, u \leq U, u++$ **do**
  $\boldsymbol{H}_t \leftarrow \sum_{i=1}^{L} \gamma_i^{t^*} e^{j\theta_{iu}} \boldsymbol{H}_t^i$
  $\boldsymbol{\psi}_t[u] \leftarrow ||\boldsymbol{H}_t||^2$;
u_t^* = max($\boldsymbol{\psi}_t$);
return $\boldsymbol{\Phi}_{\boldsymbol{r}}^* = \boldsymbol{\Phi}_{u_r^*}^r$ and $\boldsymbol{\Phi}_{\boldsymbol{t}}^* = \boldsymbol{\Phi}_{u_t^*}^t$.

the transmit antenna to the receiver in the T-space for the u^{th} realization is given by $\boldsymbol{H}_t(u) = \sum_{i=1}^{L} \gamma_i^{t^*} e^{j\theta_{iu}} \boldsymbol{H}_t^i$. We select the transmission coefficient vector among the U realizations to be the optimum one which gives us maximum Forbenius norm of the effective channel matrix from the transmitter to the receiver in the T-space. Mathematically, the selected transmission coefficient realization or configuration, denoted by u_t^* is given by $u_t^* = \text{argmax}_u \ ||\boldsymbol{H}_t(u)||^2$, where $u \in \{1, 2, \ldots, U\}$ and the corresponding STAR IRS transmission coefficient is $\boldsymbol{\Phi_t^*} = \boldsymbol{\Phi}_{u_t^*}^t$. The above algorithm is summarised in Algorithm 2. The computational complexity of this algorithm can be explained based on the following steps:

- For each of the randomly generated phase vector and magnitude vector for reflection coefficient, computation of $\boldsymbol{H}_r$ involves L complex additions. So, the time complexity is given by $\mathcal{O}(UL)$.
- For each randomly generated phase vector, to get the optimal reflection coefficient, we need to find u_r^* by taking max of $\boldsymbol{\psi}_r$ respectively, which is an U length vector. Hence, the time complexity associated with this step is $\mathcal{O}(U)$.
- With optimal magnitude vector of reflection coefficient, we calculate the optimal magnitude vector for transmission coefficient by performing L operations of (11), which has additional $\mathcal{O}(L)$ complexity.
- For each randomly generated phase vector, computation of $\boldsymbol{H}_t$ involves L complex additions. So, the time complexity is given by $\mathcal{O}(UL)$.
- To get the optimal transmission coefficient, we need to find u_t^* by taking max of $\boldsymbol{\psi}_t$, which is an U length vector. So, the time complexity is $\mathcal{O}(U)$.

The overall complexity for the algorithm will be the sum of the complexities of all these steps. This algorithm has an additional computational complexity of $\mathcal{O}(L)$ as compared to Algorithm 1.

IV. Numerical Results

In this section, we present our numerical results to illustrate the BER performance of STAR IRS-aided OTFS systems based on the two proposed algorithms. We elucidate the effect of the number of IRS elements on BER and compare its performance against conventional IRS-aided OTFS systems. BER of the STAR IRS-aided OTFS system was computed as an average of the BER obtained at users in the T and the R-space. The system parameters are listed in Table I.

Fig. 2 plots BER as a function of SNR for two aforementioned algorithms for STAR IRS-aided OTFS systems. The total path gain from the transmitter to receiver in the T and R-space may be different. One may be weak and one may be strong. Hence, splitting the incident energy equally between these two may not be a good idea. We observe that keeping L fixed at 24, the BER improves if we also optimise the magnitude of the reflection and transmission coefficient along with the phase shifts based on Algorithm 2.

TABLE I: Simulation parameters

Frame Size (M, N)	(2,2)
Carrier Frequency (f_c)	4 GHz
Sub-carrier Spacing (Δf)	3.45 kHz
DD (τ, ν) for 2 paths	$(0, 0)$), $\left(\frac{1}{M\Delta f}, \frac{1}{NT}\right)$
Maximum Speed	506.25 km/hr
Maximum Doppler Spread	1.875 kHz
Modulation	QAM-4
Detection Scheme	MMSE

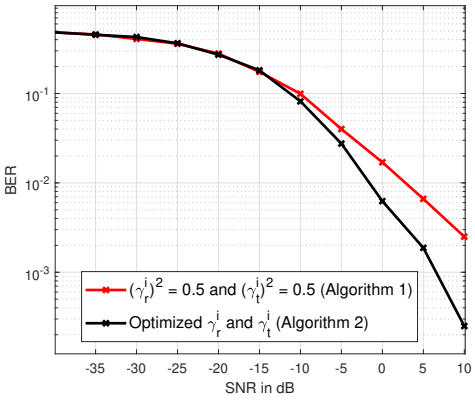


Fig. 2: Performance boost for optimized γ_T^i & γ_R^i values in STAR IRS-aided OTFS (L = 24, $U = 10^4$)

Fig. 3 plots the BER as a function of L for both the STAR IRS-aided OTFS system and the conventional IRS-aided OTFS system. For STAR IRS-aided OTFS system, we applied two step amplitude and phase programming. For a fairer comparison with conventional IRS-aided OTFS system, we assume that L/2 elements will be configured based on the receiver in T-space and rest of L/2 elements will be configured based on the receiver in R-space. An average of the BERs, calculated at these two receivers, is used to plot the graphs in Fig. 3. We observe that for a lower number of IRS elements, STAR IRS-aided OTFS system performs at par or worse than conventional IRS-aided OTFS systems. However, as we increase the number of elements, performance of STAR IRS-aided OTFS surpasses that of conventional IRS-aided OTFS system. The conventional IRS-aided OTFS system suffers a significant loss in performance due to its lesser degrees of freedom caused by their inflexibility in choosing between transmission and reflection for each element. This restriction also causes the difference in performance between conventional IRS-OTFS and STAR-OTFS to become much more noticeable as L increases.

In reality, the STAR IRS elements cannot induce continuous phase shifts on the signals incident on it. Generally, with n PIN diodes fabricated on each STAR IRS element, 2^n levels or n bit phase shift can be induced. In Fig. 4, we plot BER as a function of SNR for 1-bit, 2-bits, 3-bits and infinite precision (∞-bits) for STAR IRS-aided OTFS system using two step amplitude

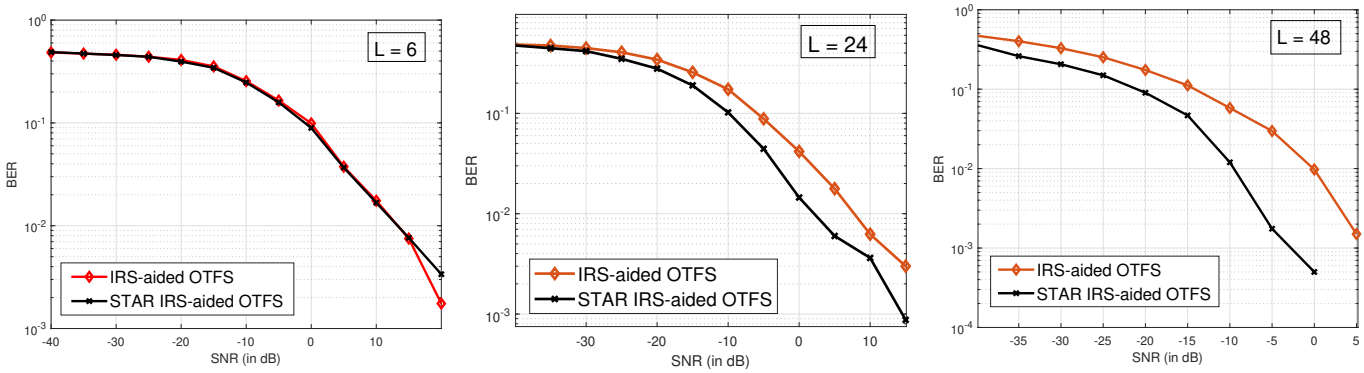


Fig. 3: Performance comparison between IRS-aided OTFS and STAR IRS-aided OTFS systems using Algorithm 2 ($U = 10^4$)

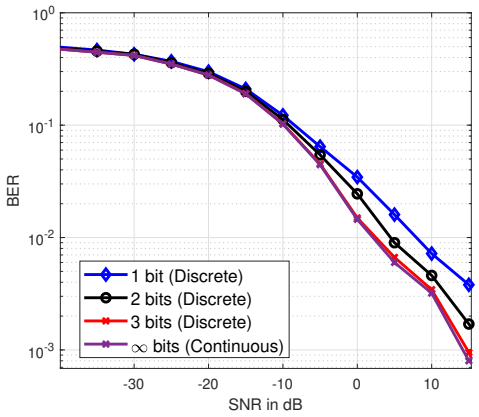


Fig. 4: Impact of discrete phase shifts on BER ($L = 24$, $U = 10^4$)

and phase programming. As the number of bits increases, the BER performance of STAR IRS-aided OTFS systems improves as reflected signals get added at the receiver more coherently. We observe that it is sufficient to have 3-bit phase shift at STAR IRS to obtain same performance as an STAR IRS offering continuous phase shift in an OTFS system.

V. Conclusions

We developed a transmit-receive relation for a novel STAR IRS-aided OTFS system in order to achieve uniform coverage in all directions. Two novel algorithms were proposed. With equal split IRS phase programming, we optimized the phase matrices corresponding to the receiver in the T and the R-spaces keeping the magnitude of the transmission and reflection coefficients equal and fixed. For two step amplitude and phase programming, we optimized both the magnitude and phases of the transmission and reflection coefficient simultaneously in the STAR IRS-aided OTFS systems. The complexity of these two algorithms was also analysed. Numerical results were presented to benchmark performance against conventional IRS-aided OTFS system and to compare the BER performance under the two algorithms. We also noticed that a 3-bit phase shift may be enough to achieve similar BER performance as an IRS with infinite phase precision. Developing lower-complexity algorithms for optimizing the magnitude and phases of the STAR IRS-aided OTFS systems, extension to multiple antennas, and analyzing the energy efficiency and latency are some interesting avenues for future research.

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