EE 632: Advanced Topics in Communications

Homework 1

Question 1. For a spreading factor SF, bandwidth B and $T_s = 2^{\text{SF}}/\text{B}$, the message symbol $s(nT_s) \in \{0, 1, 2, \ldots, 2^{\text{SF}} - 1\}$ is modulated using a frequency shift chip waveform as follows:

$$c\left(nT_s + kT\right) = \frac{1}{\sqrt{2^{\text{SF}}}} e^{\jmath 2\pi \left[(s(nT_s)+k) \bmod 2^{\text{SF}}\right] \frac{k}{2^{\text{SF}}}}, \text{ for } k = 0, 1, \ldots, 2^{\text{SF}} - 1, \tag{1}$$

where $T = 1/\text{B}$. Prove that the above waveform with $s(nT_s) = i$ and $s(nT_s) = q$ are orthogonal for $i \neq q$.

Question 2.

1. Design a LoRa system that can achieve a transmission bit rate of 6.84 Kbps whrn operating in a bandwidth of 250 KHz and coding rate of 0.5.
2. Design a LoRa system that can achieve a transmission bit rate of 0.58 Kbps whrn operating in a bandwidth of 500 KHz and coding rate of 0.8.

Question 3. Design a neural network with two boolean inputs x_1 and x_2 and two boolean outputs y_1 and y_2 as shown in the below table. Select the appropriate activation function and determine the weights and biases.

x_1	x_2	y_1	y_2
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

Question 4. Consider a sequence of independent bits $\mathbf{c} = [c_1, \ldots, c_M]$ with

$$\text{Pr}\left(c_i = 0\right) = 1 - \text{Pr}\left(c_i = 1\right) = p,$$

for $i = 1, \ldots, M$ given as input to the symbol mapper. Let $\mathbf{s} \in \mathbb{C}^{N \times 1}$ denote the corresponding symbols transmitted. Let

$$\hat{\mathbf{s}} = \mathbf{s} + \mathbf{n}$$

denote the received complex symbol vector where $\mathbf{n} \in \mathbb{C}^{N \times 1}$ denote the complex Gaussian noise vector with independent and identically distributed $\mathcal{CN}(0, \sigma^2)$ entries. Compute

$$\log_2 \left(\frac{\text{Pr}\left(c_i = 0 | \hat{\mathbf{s}}\right)}{\text{Pr}\left(c_i = 1 | \hat{\mathbf{s}}\right)} \right).$$