EE-668: Massive MIMO for 5G Communications: Design and Analysis

Assignment 2

Question 1.

1. For the scenarios given below, justify which one is fast fading, frequency selective fading or flat fading. For the given channel conditions doppler spread=200 KHz and delay spread = 2 μs.
   (a) A binary modulation has a data rate of 600 Kbps, $f_c = 1$ GHz.
   (b) A binary modulation has a data rate of 5 Kbps, $f_c = 1$ GHz.
2. If the coherence bandwidth is calculated as 100 KHz in the given radio channel of 900 MHz frequency, calculate the maximum symbol rate that can be transmitted over this channel that will suffer minimal inter symbol interference.

Question 2. Consider estimating the real zero mean scalar x from:

$$\mathbf{y} = \mathbf{h}x + \mathbf{w}$$

where $\mathbf{w} \sim N(0, \frac{N_0}{2}\mathbf{I})$ is uncorrelated with x and $\mathbf{h}$ is a fixed vector in $\mathbf{R}^n$.

1. Consider the scaled linear estimate $\mathbf{c}^t\mathbf{y}$ ($\parallel \mathbf{c} \parallel$=1):

   $$\hat{x} := a\mathbf{c}^t\mathbf{y} = (a\mathbf{c}^t\mathbf{h})x + a\mathbf{c}^t\mathbf{z}$$

   Show that the constant a that minimizes the mean square error $E[(x-\hat{x})^2]$ is equal to

   $$\frac{E[x^2]\mathbf{c}^t\mathbf{h}}{E[x^2]|\mathbf{c}^t\mathbf{h}|^2 + N_0/2}$$

2. Compute the mean square error (MSE) of the linear estimate $\hat{x}$, i.e., $E[(x-\hat{x})^2]$.

Question 3. State and prove the orthogonality principle of a minimum mean square estimate (MMSE) for a general case.

Question 4. The parameters given in table below are, velocity of the receiver (ν), d_1, d_2 are two propagation path lengths of a transmitted signal. For the carrier frequency $f_c = 60$ GHz. Calculate the following fading channel parameters. Coherence bandwidth (B_c), Coherence time (T_c), and Coherence interval (τ_c).

Note: Refer to Chapter-2 in Massive MIMO text book (Red book).

	Indoors $\lvert d_1 - d_2\rvert$=30 m	Outdoors $\lvert d_1 - d_2\rvert$=1000 m
Pedestrian ν=1.5 m/s	$B_c, T_c, \tau_c =?$	$B_c, T_c, \tau_c =?$
Vehicular ν=30 m/s	$B_c, T_c, \tau_c =?$	$B_c, T_c, \tau_c =?$

Question 5. In an L-branch diversity system the received siganl is given by,

$$y[i] = h[i]x + n[i], \quad i = 1, \ldots, L,$$

where x is transmitted symbol and $n[i], i = 1, \ldots, L$ are i.i.d circular symmetric-AWGN noise with variance σ^2. Consider a coherence receiver that linearly weights the received signals as $w_i y[i]$ and adds them to get $Z = \sum_{i=1}^{L} w_i y_i$.

1. Show that the optimum weights that maximize the instantaneous SNR (i.e not average over fading) of Z is given by

$$w[i] = h[i]^*.$$

2. What is the final expression for the instantaneous SNR? Interpret it.

Hint: Use the cauchy-schwartz inequality.