EE-668: Massive MIMO for 5G Communications: Design and Analysis

Programming assignment 1

Question 1. Generate N samples of a real Gaussian random variable (RV) X with zero mean and unit variance, i.e., $X \sim \mathcal{N}(0,1)$. Let x_i denote the i^{th} sample.

1. Plot absolute value of the sample mean $|m_N|$, where $m_N = \sum_{i=1}^{N} x_i/N$, as a function of N for $N = 1, 10, 10^2, 10^3, 10^4, 10^5, 10^6$.
2. Plot sample variance $\sum_{i=1}^{N}(x_i - m_N)^2/N$ as a function of N for $N = 1, 10, 10^2, 10^3, 10^4, 10^5, 10^6$.
3. Comment on the convergence of mean and variance.

Question 2. Plot the complementary CDF of $X \sim \mathcal{N}(0,1)$, i.e. $Q(a) = \Pr(X > a)$ and its bounds.

$$\frac{1}{\sqrt{2\pi}a}\left(1 - \frac{1}{a^2}\right)e^{-\frac{a^2}{2}} < Q(a) < e^{-\frac{a^2}{2}}, a > 1.$$

Question 3. Consider a two-dimensional Gaussian ransom vector with mean vector μ and covariance matrix $\mathbf{C}$, i.e., $\mathbf{x} \sim \mathcal{N}(\mu, \mathbf{C})$. Plot the contour plots of the probability density function of X, when.

1. $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$
2. $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}.$
3. $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}.$
4. $\mu = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$
5. $\mu = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 2 \end{bmatrix}.$

Question 4. Generate two independent Gaussian RVs X_R and X_I such that $X_R \sim \mathcal{N}(m_r, \sigma_r^2)$ and $X_I \sim \mathcal{N}(m_i, \sigma_i^2)$. Let $X = X_R + jX_I$ be a complex Gaussian RV. Compute probability density function of X, for $x = 1 + j, -0.3660 + j1.3660i, 0.3660 + j1.3660$ and compute $\mathbb{E}\left[X^2\right]$, $\mathbb{E}\left[|X|^2\right]$, and $\mathbb{E}\left[|X|^4\right]$ for the below three cases

1. $m_r = 0$, $m_i = 0$, $\sigma_r^2 = 0.5$, $\sigma_i^2 = 0.5$.
2. $m_r = 1$, $m_i = 1$, $\sigma_r^2 = 0.5$, $\sigma_i^2 = 0.5$.
3. $m_r = 0$, $m_i = 0$, $\sigma_r^2 = 1.5$, $\sigma_i^2 = 0.5$.

Comment on the observations.

List of useful MATLAB functions

a) randn
b) mean
c) var
d) sqrt
e) erfc
f) qfunc
g) contour
f) meshgrid